

Lecture 14 (10/11/2021)

* Prayer

* Spiritual thought ...

* Nonhomogeneous linear ODE of 2nd order:

$$y'' + p(t)y' + q(t)y = g(t) \rightsquigarrow y'' + py' + qy = g \quad (*)$$

We will see that if we can solve the homogeneous equation

$$y'' + p(t)y' + q(t)y = 0 \quad (**)$$

then we can solve (*) for any function g .

Observation :

(1) If y_0 is a solution to (*) and \tilde{y} is a solution to (**) then $y_0 + \tilde{y}$ is a solution to (*).

(2) If we can find one solution to (*) and all solutions to (**) then we get all solutions to (*).

$$y = y_p + y_c$$

↑ general sols to (**)
a particular sol to (*)

The problem of solving for all sols of (*) becomes the problem of finding all sols of (**) and one sol. of (*).

If g is in some special forms, one can find a particular sol. to (*) easily.

Ex $y'' + 3y' + 2y = t$

test candidate $y = At$ $\rightsquigarrow 0 + 3A + 2At = t \rightsquigarrow \begin{cases} A = \frac{1}{2} \\ A = 0 \end{cases}$ (not work!)

Test another candidate $y = At + B \rightsquigarrow 0 + 3A + 2(At + B) = t$

$$\rightsquigarrow \begin{cases} 2A = 1 \\ 3A + 2B = 0 \end{cases} \rightsquigarrow \begin{cases} A = \frac{1}{2} \\ B = -\frac{3}{4} \end{cases}$$

This works! The method we just used is called "undetermined coefficients method".

More generally, if we are given the ODE $ay'' + by' + cy = P(t)$, where P is a polynomial then we can test the candidate $y = Q(t)$ where Q is of the same degree as P , with coefficients TBD.

* More examples:

$$\textcircled{1} \quad y'' + 3y' + 2y = e^{2t}$$

Test candidate $y = A e^{2t}$.

$$\textcircled{2} \quad y'' + 3y' + 2y = \sin 3t$$

Test candidate $y = A \cos 3t + B \sin 3t$.

$$\textcircled{3} \quad y'' + 3y' + 2y = t e^{2t}$$

Test candidate $y = (At+B) e^{2t}$.

$$\textcircled{4} \quad y'' + 3y' + 2y = t + e^t + \sin t$$

Split:

$$y'' + 3y' + 2y = t$$

$$y'' + 3y' + 2y = e^t$$

$$y'' + 3y' + 2y = \sin t$$

} Solve three eqs. independently of each other. Get y_1, y_2, y_3

then $y = y_1 + y_2 + y_3$.

This is known as the superposition principle.