Lecture 16

Friday, October 8, 2021 2:59 AM

* Prayer

* Spiritual thought ....

* An example of last lecture:

$$y'' - 2y' + y = te^t$$

Given $y = (At+B)e^t$

$\longrightarrow \quad y = t(At+B)e^t$

$\longrightarrow \quad y = t^2(At+B)e^t$ ✓

$$A = \frac{1}{6}, \quad B = 0$$

Demonstrate on Mathematica ....

* Variation of parameters:

$$y'' + p(t)y' + q(t)y = g(t) \qquad (*)$$

Suppose we have two sols to the homogeneous version: y_1, y_2

$y = c_1 y_1 + c_2 y_2$ is a general sol to the homo. ver.

To search for a sol. to $(*)$, we allow c_1, c_2 to be functions of t.

$$y = u_1 y_1 + u_2 y_2$$

$q(t) \times \quad y = u_1 y_1 + u_2 y_2$

$p(t) \times \quad y' = \underbrace{(u_1' y_1 + u_2' y_2)}_{\text{forced to be } 0} + (u_1 y_1' + u_2 y_2')$

$1 \times \quad y'' = (u_1' y_1' + u_2' y_2') + (u_1 y_1'' + u_2 y_2'')$

$$g(t) = u_1' y_1' + u_2' y_2' + u_1 \underbrace{(y_1'' + p y_1' + q y_1)}_{0} + u_2 \underbrace{(y_2'' + p y_2' + q y_2)}_{0}$$

In summary:

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = g \end{cases}$$

$$\rightsquigarrow \quad u_1' = \frac{\begin{vmatrix} 0 & y_2 \\ g & y_2' \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}, \qquad u_2' = \frac{\begin{vmatrix} y_1 & 0 \\ y_1' & g \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}$$

Then integrate to get u_1, u_2. Then $y = u_1 y_1 + u_2 y_2$ is the general sol to $(*)$.

<u>Ex</u> $\quad y'' + 4y' + 4y = t^{-2} e^{-2t} = \dfrac{e^{-2t}}{t^2}$

$y_1 = e^{-2t}, \quad y_2 = t e^{-2t}, \quad W = e^{-4t}$

$$u_1' = -\frac{1}{t}, \quad u_2' = \frac{1}{t^2}$$

$$\rightsquigarrow u_1 = -\ln t + c_1, \quad u_2 = -\frac{1}{t} + c_2$$

$$y = u_1 y_1 + u_2 y_2 = (-\ln t + c_1) e^{-2t} + \left(-\frac{1}{t} + c_2\right) t e^{-2t}$$

$$= -(\ln t) e^{-2t} + c_3 e^{-2t} + c_2 t e^{-2t}$$