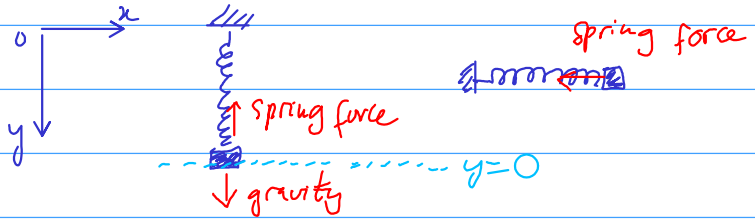
Lecture 17 (16/11/2021)

* Prayer
* Spiritual thought....

* Models with 2nd order linear ODE : mechanical and electrical vibration.



Write an equation that describes the motion of the spring.

Step 1 : write Newton's 2nd law $F = ma$.

F : net force, a : acceleration

$a = y''$

$F = \underbrace{mg}_{\text{gravity force}} + \underbrace{F_s}_{\text{spring force}} + \underbrace{F_d}_{\text{damping force}}$

Spring force $= -k(L+y)$ (Hooke's law)

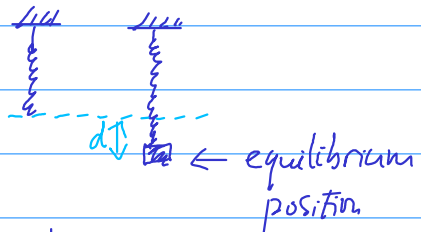
Damping force $= -\gamma y'$

Thus, $my'' = mg - k(L+y) - \gamma y'$.

Note : $mg = kL$.

$\rightsquigarrow my'' + \gamma y' + ky = 0$.

Ex :



$d = 2 \text{ in} = \frac{1}{6} \text{ ft}$

w (weight) $= 2$ lbs

$\rightsquigarrow m = \frac{w}{g} = \frac{2}{32} = \frac{1}{16} \underbrace{(\text{lbs} \cdot \text{s}^2/\text{ft})}_{\text{slug}}$

Resistence $= \frac{1}{8}$ lbs when the velocity is $1 \text{ in/s} = \frac{1}{12}$ ft/s

$\rightsquigarrow \gamma \cdot \frac{1}{12} = \frac{1}{8}$

$\rightsquigarrow \gamma = 3/2$

$mg = w = kd$

$\rightsquigarrow k = \frac{w}{d} = \frac{2}{1/6} = 12$

Put all together: $\frac{1}{16}y'' + \frac{3}{2}y' + 12y = 0$

$\rightsquigarrow \quad y'' + 24y' + 192y = 0$

$\rightsquigarrow \quad y = e^{-12t}\left(c_1 \cos(4\sqrt{3}\,t) + c_2 \sin(4\sqrt{3}\,t)\right)$

↑ quasi-frequency