Lecture 19 (10/15/2021)

* Prayer
* Spiritual thought
* Last time:

$$\frac{1}{4}y'' + 2y' + 16y = 8\sin 8t. \qquad (*)$$

Initial conditions: $y(0) = -1/4$, $y'(0) = 0$.

Get

$$y = \underbrace{-\frac{1}{2}\cos 8t}_{\text{steady state sol.}} + e^{-4t}\left(\frac{1}{4}\cos 4\sqrt{3}t + \frac{\sqrt{3}}{12}\sin 4\sqrt{3}t\right)$$

Question (from control theory perspective): what frequency of the external force gives the largest vibration (of the steady state solution)? (resonance)

$$F = 8\sin\omega t$$

$$y = y_c + y_p$$

Steady state: $y_p = A\cos\omega t + B\sin\omega t \longrightarrow$ determine A, B as functions of ω.

Amplitude of steady state sol $= \sqrt{A^2+B^2}$

$$= \frac{32}{\sqrt{(8\omega)^2 + (\omega^2-64)^2}}$$

$\longrightarrow$ maximum when $\omega^2 = 32 \rightarrow \omega = 4\sqrt{2}$

In general,

$$\omega_{max} = \begin{cases} \sqrt{\frac{k}{m}}\sqrt{1-\frac{\gamma^2}{2mk}} & \text{if } \gamma^2 < 2mk \\ 0 & \text{if } \gamma^2 \geq 2mk \end{cases}$$

Special case: $\gamma = 0$, $\omega_{max} = \sqrt{\frac{k}{m}} = \omega_0$: the natural frequency

Linear ODE of order n:

$$y^{(n)} + p_1(t)\,y^{(n-1)} + p_2(t)\,y^{(n-2)} + \dots + p_{n-1}(t)\,y' + p_n(t)\,y = g(t)$$

Initial conditions:

$$y(t_0) = y_0,\quad y'(t_0) = y_0',\ \dots,\ y^{(n-1)}(t_0) = y_0^{(n-1)}$$

* Abstract theorem:

The IVP has a unique sol. on interval I containing t_0 on which $p_1, \dots, p_n$ are continuous.