Lecture 2

Monday, August 30, 2021 1:43 PM

* Prayer
* Spiritual thought

By small and simple things, great things are brought to pass.
The Lord gives us patterns in all things

* Classifications of DEQ:

- Ordinary vs partial
- order
- Linear vs Nonlinear
- autonomous vs nonautonomous

* Examples:

(1) $P(t)$ = population at time t.

Each individual has a birth rate of c. That is, an individual gives birth to ct individuals after time t.

$$1 \to 1 + c\,dt \quad \text{after time } dt$$

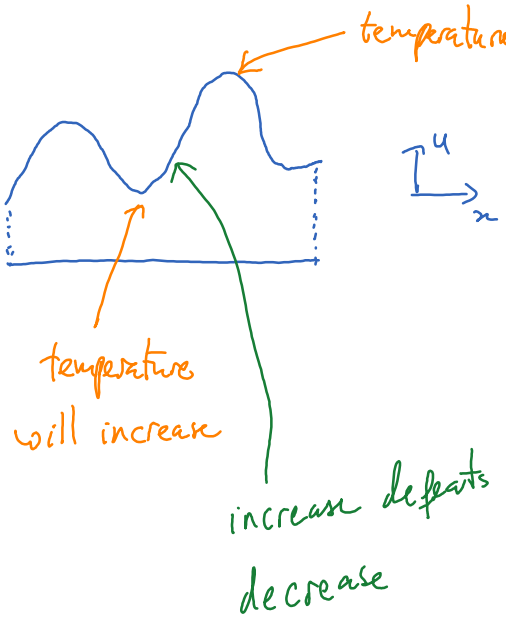
$$P(t) \to (1 + c\,dt)P(t) \quad \text{after time } dt$$

Thus, $P(t+dt) = (1 + c\,dt)P(t) \rightsquigarrow P'(t) = cP(t)$

(2)

$u(x,t)$ = temperature at position x at time t.

At a fixed time t:



"curvature"

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial x^2}$$

heat equation

(3) Equations of the form $y' = f(t,y)$.

f is a given function

Intuition: if we know the value of y at time t_0, then we also know the rate of change of y at that time. The population model is an example. The falling object problem is another example ($u' = g - cu$).

* Drawing direction fields using Mathematica...

VectorPlot[{1, f(t,y)}, {t,-1,1}, {y,-1,1}]

StreamPlot[.....]

Example:

$y' = 2t - y$, $y(0) = 1$

Asymptotic behavior of y?

* How to solve for the equation $y' = 2t - y$?

Take a simpler equation: $y' = 2 - y$

$$\frac{dy}{dt} = 2 - y \rightsquigarrow \frac{dy}{2-y} = dt \rightsquigarrow \cdots$$