# Lecture 5

Thursday, September 9, 2021 9:28 PM

* Prayer

* Spiritual thought : continue to find tools to solve differential equations.

- Integrating factor : the idea is not restricted on $1^{st}$ order linear ODEs, for example

$$y'' + 2y' + y = 1$$

Multiply both sides by $e^t$ : $(e^t y)'' = e^t$

Then integrate .....

[the method of finding the integrating factor is, in general, tricky.]

- Separation of variables : (not restricted to ODE of $1^{st}$ order or linear).

$$y' = \underbrace{f(x,y)}_{= h(x) r(y)} \qquad (*)$$

Ex

$$y' = xy \qquad y' = \frac{1+y^2}{1+x^2} \qquad \cancel{y' = x+y}$$

$$y' = \frac{y}{e^x} \qquad y' = \frac{1+x^2}{y}$$

Note that $y'$ is also written as $\frac{dy}{dx}$.

$(*)$ can be written as $\frac{dy}{r(y)} = h(x)\,dx$.

Method: integrate both sides. Sometimes $y$ can be solved as an explicit function of $x$.

Ex: $y' = \frac{2x}{y+1}$, $y(0) = 1$

$\rightsquigarrow (y+1)y' = 2x$

$\rightsquigarrow \int (y+1)y'\,dx = \int 2x\,dx$

$\rightsquigarrow \frac{y^2}{2} + y = x^2 + C$

Substitute $x=0$: $C = \frac{3}{2}$

$\rightsquigarrow y^2 + 2y = 2x^2 + 3 \rightsquigarrow \boxed{y = -1 + \sqrt{2x^2+4}}$

Ex $y' = \frac{-2x}{y^3+1}$ $\rightsquigarrow (y^3+1)y' = -2x$

$\rightsquigarrow \int (y^3+1)\,dy = -\int 2x\,dx$

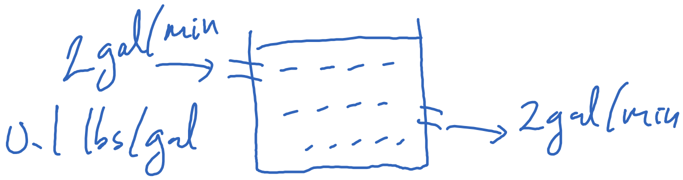
$\rightsquigarrow \frac{y^4}{4} + y + x^2 = C$

Use Mathematica to draw the curves: ContourPlot

$y(-1) = 0$. Where is maximum attained? Where does the solution cease to exist?

# Modelling

* The mixing problem:




$V_0 = 500\text{gal}$, $y_0 = 5\text{ lbs}$

$y(t) = ?$

$$y'(t) = \text{salt in} - \text{salt out} = 2 \times 0.1 - \frac{y}{V(t)} = 0.2 - y$$

$$\begin{cases} y' = 0.2 - y \\ y(0) = 0.01 \end{cases}$$

Two ways to solve the problem
- Integrating factor
- separation of variables.

- Reading assignment: compound interest (Example 2, p. 43)