Lecture 7

Tuesday, September 14, 2021 11:15 PM

* Prayer
* Spiritual thought

* Recall: $y' = f(t,y)$, $y(t_0) = y_0$ $(*)$

[If f and f_y are cont. on a rectangular box containing point (t_0, y_0) then $(*)$ has a unique sol. on the interval $(t_0 - h, t_0 + h)$ for some $h > 0$.]

Application: by continuing this argument, one can show that the solution exists and is unique on some maximal time-interval.

* Special case: If f is linear in y, i.e. $f(t,y) = a(t)y + b(t)$, then the sol. exists on the interval where $a(t)$ and $b(t)$ are both continuous.

Ex $(\sin t)y' + (\ln(t-1))y = t$

$$\rightsquigarrow y' + \underbrace{\frac{\ln(t-1)}{\sin t}}_{\text{cont. on } (1,\pi), (\pi, 2\pi), \ldots} y = \frac{t}{\sin t} \Big\} \text{cont. on } (0,\pi), (-\pi, 0), (-2\pi, -\pi), (\pi, 2\pi) \ldots$$

Solution exists on $\underbrace{(1,\pi),\ (\pi,2\pi),\ (2\pi,3\pi),\ \ldots}_{\text{guaranteed}}$

If the initial condition $y(2)=1$ is specified, then the sol is unique on the interval $(1,\pi)$.

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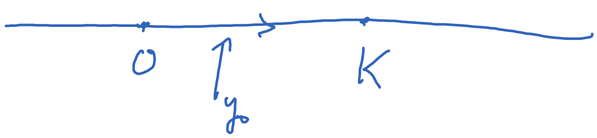
* Population model revisited:

- $\underbrace{y' = ry}_{\text{good for only a short time}}$ $\qquad r$: birth rate

- r should depend on y or t.

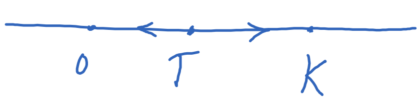
$$\underbrace{y' = r\left(1-\frac{y}{K}\right)y}_{\text{logistic equation}}, \quad y(t_0)=y_0$$

K: saturation level/ carrying capacity.



This model is not a good practical model when there are only very few

- $y' = -ry\left(1-\frac{y}{T}\right)\left(1-\frac{y}{K}\right)$ $\qquad T$: threshold level.

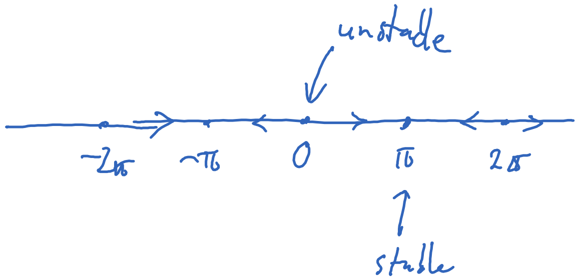


Perhaps it's better to introduce autonomous ODE, equilibrium states, asymptotic stability before going to the population model.

Ex

$y' = \sin(y)$ $\qquad$ $y' = y e^{y}$ $\qquad$ ....

$y' = y\sin(y)$ $\qquad$ $y' = y^2$ $\qquad$ .....



phase line