Lecture 8

Thursday, September 16, 2021 11:00 PM

* Prayer

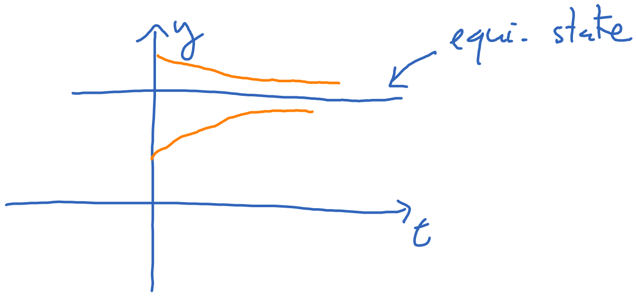
* Spiritual thought

* Autonomous ODE $y' = f(y)$:

- Equilibrium states are the "zeros" of f. If the solution starts at an equilibrium, it stays at the equilibrium all the time.
- There are 3 types of equilibrium states:

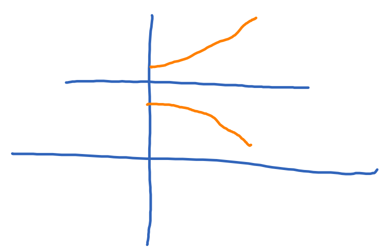
$$\begin{cases} \text{Stable} \\ \text{unstable} \\ \text{Semistable} \end{cases}$$

Graphically,

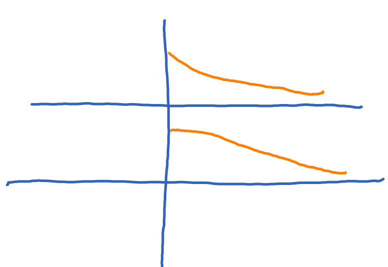


stable

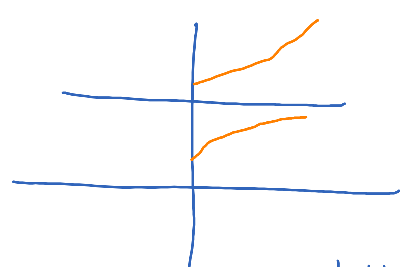
"yt"-diagram



unstable



Semistable

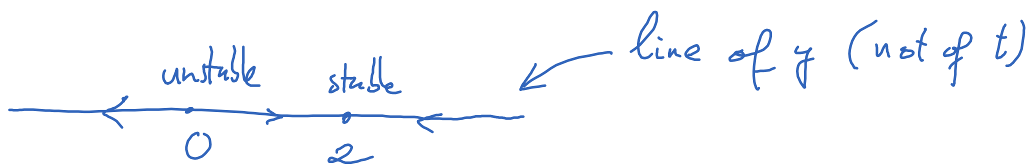


semistable

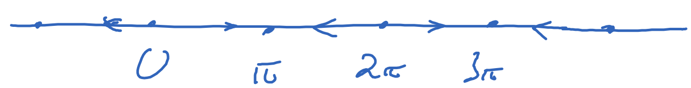
- A solution that starts between two consecutive equilibrium states remain trapped in between those states.

Phase line

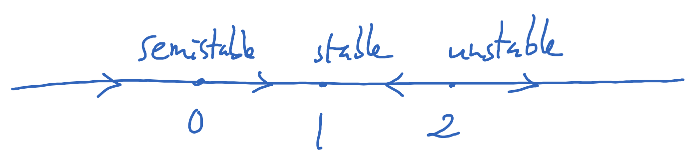
$$y' = y(2-y)$$



$$y' = \sin(y)$$



$$y' = y^2(y-1)(y-2)$$



* Population model revisited :

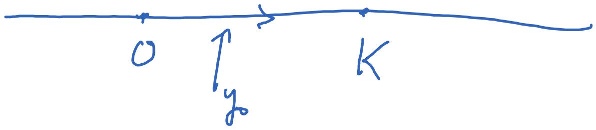
- $\underbrace{y' = ry}_{\text{good for only a short time}}$ r : birth rate

- r should depend on y or t.

$$y' = r\left(1 - \frac{y}{K}\right) y \quad , \quad y(t_0) = y_0$$

logistic equation

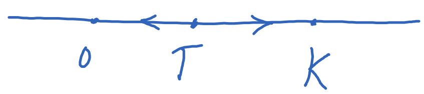
K : saturation level / carrying capacity.



This model is not a good practical model when there are only very few

- $$y' = -ry\left(1 - \frac{y}{T}\right)\left(1 - \frac{y}{K}\right)$$

T : threshold level.



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* In our ODE toolbox:

- Integrating factor : multiply both sides by a suitably chosen factor so that we can integrate.
- Separation of variables : split the variables on two sides so that we can integrate.
- Graphic methods for $y' = f(t,y)$: direction fields, phaseline.
- $\boxed{\text{Exact differential equation}}$: use the chain rule of differentiation.

<u>Ex</u> : $xy' + y = 1 \rightsquigarrow (xy)' = 1$

$$x + y'\sin y = 1 \rightsquigarrow \left(\frac{x^2}{2} - \cos y\right)' = 1 \rightsquigarrow \frac{x^2}{2} - \cos y = x + C$$

Generally, we deal with an ODE of the form

$$\left.\begin{aligned} &\underbrace{M(x,y)}_{\frac{\partial \phi}{\partial x}} + \underbrace{N(x,y)}_{\frac{\partial \phi}{\partial y}}y' = 0 \quad (*) \end{aligned}\right\} \text{exact differential form}$$

want:

so that $(*)$ becomes $\dfrac{d}{dx}\phi(x,y(x)) = 0$

$$\rightsquigarrow \phi(x,y) = C$$

Ex: $\underbrace{(3x^2 - 2xy + 2)}_{M} + \underbrace{(6y^2 - x^2 + 3)}_{N}y' = 0$

Test: $\left.\begin{aligned} M_y &= -2x \\ N_x &= -2x \end{aligned}\right\}$ in exact diff. form