Lecture 9

Saturday, September 18, 2021 11:59 PM

* Prayer

* Spiritual thought

* In our ODE toolbox:

- Integrating factor: multiply both sides by a suitably chosen factor so that we can integrate.

- Separation of variables: split the variables on two sides so that we can integrate.

- Graphic methods for $y' = f(t, y)$: direction fields, phaseline.

- $\boxed{\text{Exact differential equation}}$: use the chain rule of differentiation.

Ex:

$$xy' + y = 1 \rightsquigarrow (xy)' = 1$$

$$x + y'\sin y = 1 \rightsquigarrow \left(\frac{x^2}{2} - \cos y\right)' = 1 \rightsquigarrow \frac{x^2}{2} - \cos y = x + C$$

$$y + 2x + y'x = 1 \rightsquigarrow (yx + x^2)' = 1 \rightsquigarrow yx + x^2 = x + C$$

Generally, we deal with an ODE of the form

$$\underbrace{M(x,y)}_{} + \underbrace{N(x,y)}_{} y' = 0 \quad (*)$$

want: $\dfrac{\partial \phi}{\partial x}$ $\quad \dfrac{\partial \phi}{\partial y}$

} exact differential form

so that $(*)$ becomes $\dfrac{d}{dx} \phi(x, y(x)) = 0$

$$\rightsquigarrow \phi(x,y) = C$$

Ex: $\underbrace{(3x^2 - 2xy + 2)}_{M} + \underbrace{(6y^2 - x^2 + 3)}_{N} y' = 0$

Test: $M_y = -2x$, $N_x = -2x$ } in exact diff. form

ODE of second order: $F(t, y, y', y'') = 0$.

Linear ODE of second order:

$$y'' + p(t) y' + q(t) y = g(t)$$

$g(t) \equiv 0$: homogeneous

$g(t) \not\equiv 0$: nonhomogeneous

We need two initial conditions to get a unique solution. (In theory, we have to integrate twice to get y, which then gives us two undetermined constants to find. One way is $y(t_0) = y_0$, $y'(t_0) = y_0'$.

An interesting feature of linear ODE is that one can split it into simpler ODEs and then solve them independently.

<u>Ex</u> $$y' + xy = x + e^{x} + \sin x$$

$$\left.\begin{aligned} &\to y_1' + xy_1 = x \\ &\to y_2' + xy_2 = e^{x} \\ &\to y_3' + xy_3 = \sin x \end{aligned}\right\} \; y = y_1 + y_2 + y_3$$

In general, there are no general methods that can solve <u>any</u> 2^{nd} ODE.

However, if the ODE has only constant coefficients then we can solve for the solution.

<u>Ex</u> $$y'' + 3y' + 2y = 0$$

$$y'' + 2y' + y = 0$$

$$\cdots\cdots$$

Method: try $y = e^{rt}$ where r is a constant to be determined.