Midterm I: Some problems for review

The first midterm will take place at the Testing Center during Oct 6-8. It is a closed-book exam, 2 hours long. *Beware that they are closed at 10 PM on Wednesday, Thursday, and 7 PM on Friday.* You are allowed to use a non-graphing calculator. The covered sections are 1.1 - 3.4. Some techniques of solving ODEs that you have learned:

- Integrating factor
- Separation of variables
- Exact differential equation
- Testing candidates
- Direction fields
- Phase line

Some problems you should know how to do:

- Solve ODEs of the form $y' + p(x)y = g(x)$ (and variations of it).
- Solve ODEs of the form $y' = f(x)h(y)$ (and variations of it).
- Solve ODEs of the form $ay'' + by' + cy = 0$ where a, b, c are constants.
- Identify equilibrium states, classify them, draw a phase line, "ty" diagram of ODEs of the form $y' = f(y)$.
- Match direction fields with the corresponding ODEs.
- Find the Wronskian of two given functions.
- The mixing problem

Circle all correct answers. In the exam, you will not be asked to give explanations. But you should know the reason for your answers in this practice.

(1) The ODE $y' = 2y + 1$ is

Linear Nonlinear

(2) The ODE $y' = 2y + t$ is

Autonomous Non-autonomous

(3) The ODE $\frac{y'y''}{y'''} = \sin(ty)$ is

a. First order

b. Second order

c. Third order

(4) Every solution to the ODE $y' = \sin(y)$ is bounded. (That is, there is some number $M > 0$ such that $-M \leq y(t) \leq M$ for all t.)

True False

(5) Every solution to the ODE $y' = 2y^3 - 3y$ is bounded.

True False

(6) The function $y = \sin(t)$ satisfies the first order ODE $y' + ay = 0$ for some constant a.

True False

(7) Every solution to the second order ODE $y'' + y = 0$ is periodic. (That is, there is some number $T > 0$ such that $y(t + T) = y(t)$ for all t.)

True False

(8) If a function $f(t, y)$ and its partial derivative $\frac{\partial f}{\partial y}$ are continuous in the rectangle $(-1, 1) \times (0, 2)$ then the initial value problem $y' = f(t, y)$, $y(0) = 1$ has a solution on $(-1, 1)$.

True False

(9) The functions $y_1 = e^t$ and $y_2 = t^2$ can solve the same second order ODE of the form $y'' + p(t)y' + q(t)y = 0$ on the interval $(-1, 1)$.

True False

(10) The function $c_1 e^{-2t} + c_2 t e^{-2t}$ is a general solution of the ODE

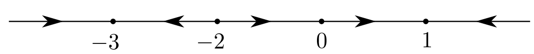
a. $y'' + 2y' + y = 0$

b. $y'' + 4y' + 4y = 0$

c. $y'' + 3y' + 2y = 0$

d. $y'' + 4y' + 4y = t$

(11) An autonomous ODE $y' = f(y)$ has the following phase line. Suppose $y(2) = -1$. Find $\lim_{t\to\infty} y(t)$.



(12) Let $u(t) = t^2$ and $v(t) = e^t$. Find $W[u, v](1)$.

(13) Consider the autonomous ODE $y' = y^2(1-y^2)(y^2-4)$. Determine all the equilibrium states and classify them (stable/unstable/semi-stable). Draw the phase line. Sketch a few approximate solutions in the ty-plane.

(14) A container initially has 500 gal of mixture of salt and water with salt concentration 0.1 lbs per gal. People purify the container by pumping fresh water into the container at a volume rate of 3 gal/min, and at the same time pumping out well-stirred liquid from the container at the rate of 2 gal/min. How much time will elapse before the concentration of the salt in the container is equal to an eighth of the original concentration?

(15) Solve the initial value problem

$$x(y' + 2) = y, \quad y(1) = 2.$$