Problem 5 of 5.3

Wednesday, October 27, 2021 1:47 PM

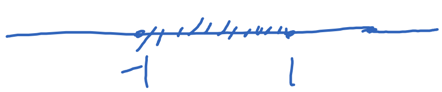
When does the power series method work?

$$\left.\begin{aligned} & y^{(n)} + p_1(x) y^{(n-1)} + \cdots + p_n(x) y = g(x), \\ & y(x_0) = y_0, \ldots, y^{(n-1)}(x_0) = y_0^{(n-1)}. \end{aligned}\right\} \quad (*)$$

If $p_1, p_2, \ldots, p_n, g$ are analytic at x_0 then $(*)$ has a unique analytic solution $y = \sum a_n (x - x_0)^n$.

An elementary function (polynomial, fractional, exponential, logarithmic, trigonometric and the combinations) is analytic at any point where it is continuous.

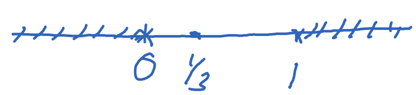
Ex: $\ln(x^2 - 1)$ is analytic at any $x_0 < -1$ or > 1.



radius of conv. $= \min\{|x_0 - 1|, |x_0 + 1|\}$

Ex: $x \ln(1-x)\, y' + e^x y = \sqrt{x}$, $y(\frac{1}{3}) = 1$.

$$\rightsquigarrow \quad y' + \underbrace{\frac{e^x}{x \ln(1-x)}}_{\substack{\text{analytic} \\ \text{at } x_0 = 1/3}} y = \underbrace{\frac{\sqrt{x}}{x \ln(1-x)}}_{\substack{\text{analytic} \\ \text{at } x_0 = 1/3}}$$



$y = \sum a_n (x - \frac{1}{3})^n \rightsquigarrow$ radius of convergence $\geq \frac{1}{3}$.