Hints for Problem 8 and 21 of 6.1

For Problem 8, you use the formula provided right above the problem: $\sin(bt) = \frac{1}{2i}(e^{ibt} - e^{-ibt})$. While trying to compute the Laplace transform of this function, you will encounter the integral $\int_0^\infty e^{ibt}e^{-ts}dt$. The integrand can by written as $e^{t(ib-s)}$. Because you are integrating with respect to t, you can treat $ib-s$ as a constant. So you can find an antiderivative of $e^{t(ib-s)}$. Once you get the antiderivative, note that $\lim_{t\to\infty} e^{t(ib-s)} = 0$. This is because $e^{t(ib-s)} = e^{-ts}e^{itb} = e^{-ts}(\cos bt + i \sin bt)$, which is a product of a decaying function e^{-ts} (remember $s > 0$) and a bounded (sinusoidal) function.

For Problem 21, note that the improper integral $\int_1^\infty t^{-2}e^t dt$ converges/diverges if and only if the "tail" integral $\int_M^\infty t^{-2}e^t dt$ converges/diverges (where M is any large number). Intuitively, when M is large, t in the integrand is large and the integrand itself is also large (say, larger than t). This hints that integral diverges. You need to make this hand-waving argument more precise in your homework.

You can double check your computation of Laplace transform with Mathematica. For example, let f be a piecewise function given by $f(t) = 1$ if $t < 1$, and $f(t) = 2$ if $t \geq 1$. Then you use the commands:

```
f[t_]:=Piecewise[{{1,t<1},{2,t>=1}}]
LaplaceTransform[f[t],t,s]
```