Lecture 13

Wednesday, May 25, 2022 1:52 AM

* Prayer

* Spritual thought

<u>Double integral:</u>

$$\iint_{[a,b]\times[c,d]} f(x,y)\,dA = \text{volume of the solid under the graph of } f$$

and above the rectangle $[a,b]\times[c,d]$

$$\left.\begin{aligned} &= \int_a^b \int_c^d f(x,y)\,dy\,dx \\ &= \int_c^d \int_a^b f(x,y)\,dx\,dy \end{aligned}\right\} \text{Fubini's theorem: the order of integration can be swapped.}$$

Ex:

$$\iint_{[0,1]\times[1,3]} \frac{x}{(1+xy)^2}\,dA = \int_0^1 \int_1^3 \frac{x}{(1+xy)^2}\,dy\,dx$$

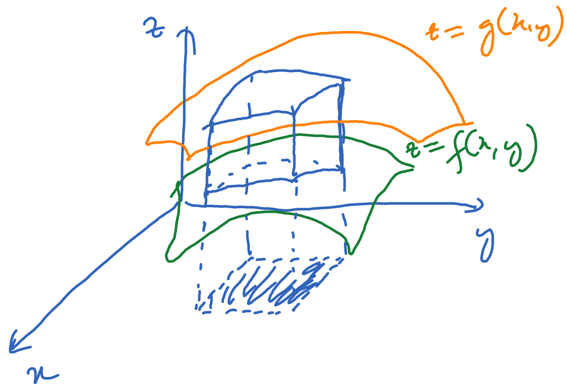
$$= \int_0^1 -\frac{1}{1+xy}\Big|_{y=1}^{y=3}\,dx$$

$$= \int_0^1 \left(\frac{1}{1+x} - \frac{1}{1+3x}\right) dx$$

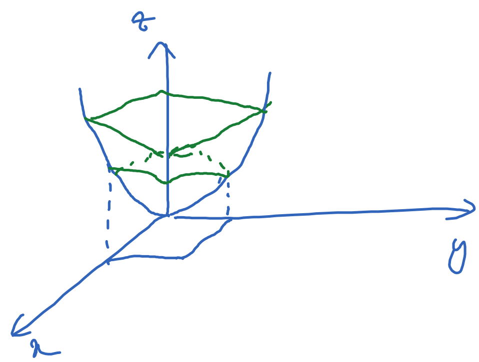
$$= \left[\ln(1+x) - \frac{1}{3}\ln(1+3x)\right]_0^1 = \ln 2 - \frac{1}{3}\ln 4 = \frac{1}{3}\ln 2.$$

The volume of the solid above a rectangle $[a,b]\times[c,d]$ and between the surfaces $z=f(x,y)$ and $z=g(x,y)$ is

$$\iint_{[a,b]\times[c,d]} \big(g(x,y)-f(x,y)\big)\,dA$$



Ex Find the volume of the solid between $z=x+y+1$ and $z=x^2+y^2$ where $(x,y)\in[0,1]\times[0,1]$.



$$\iint_{[0,1]\times[0,1]} (x+y+1-x^2-y^2)\,dA = \frac{4}{3}$$

In Mathematica, use the command Integrate:

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Integrate[ x+y+1-(x^2+y^2), {x,0,1}, {y,0,1}]
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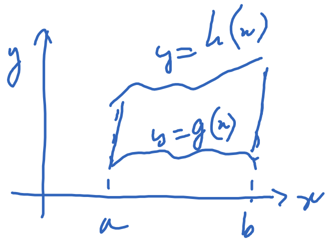
Double integral over a general region:

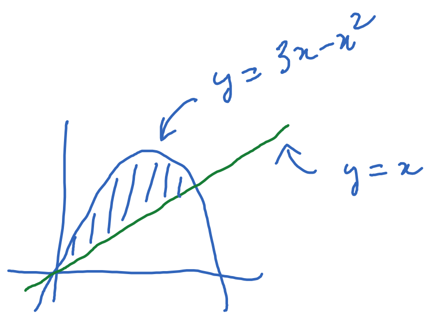
Type 1



$R = \{(x,y) \mid a \le x \le b,\ g(x) \le y \le h(x)\}$

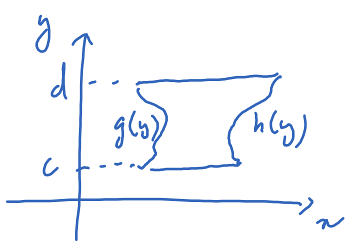
$$\iint_R f(x,y)\,dA = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$$

Ex:



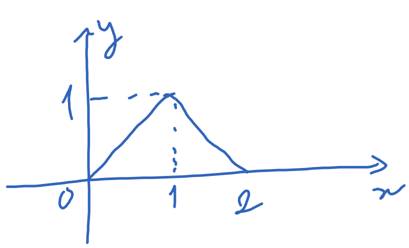
$$\iint_R 2y\,dA = ?$$

Type 2



$$\iint_R f(x,y)\,dA = \int_c^d \int_{g(y)}^{h(y)} f(x,y)\,dx\,dy$$

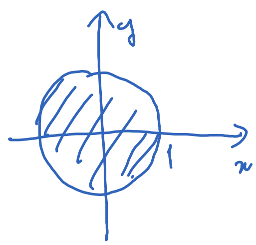
Ex:



$$\iint_R (x+y)\,dA = ?$$

Although the method above works for many regions other than the rectangles, it is still not very good for regions that have a circular part such as the circle.

Ex



$$\iint_R f(x,y)\,dA = \underbrace{\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx}$$

this integral can be difficult to deal with because of the square root.

Polar coordinates provide a way to integrate over the circle (or generally, regions that has a circular part).