Lecture 3

Monday, May 2, 2022 1:00 PM

* Prayer
* Spiritual thought

Applications of cross product

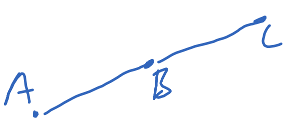
1) Find a normal vector of a plane.

$v = \overrightarrow{AB}$

$w = \overrightarrow{AC}$

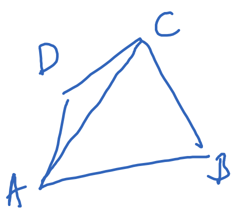
$v \times w$ is a vector orthogonal to the plane (normal vector of the plane)

2) Check if three points are on the same line (colinear).



$\overrightarrow{AB} \times \overrightarrow{AC} = 0$

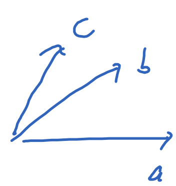
3) Find area of a polynomial



$$\text{area} = \text{area}(ABC) + \text{area}(ACD)$$

$$= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| + \frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{AD}|$$

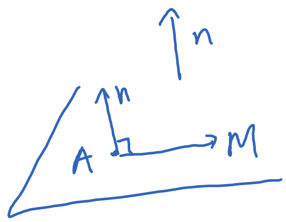
4) Check if 4 points are coplanar:
(or 3 vectors)



$a \cdot (b \times c) = 0$

Ex Find x such that 4 points $A(-1,0,1)$, $B(2,1,1)$, $C(-2,2,-1)$ and $D(x,1,3)$ are coplanar.

Equation of a plane



A plane is uniquely determined if we know a point plane passes through and a normal vector.

The plane consists of all points M such that

$$\vec{AM} \perp n.$$

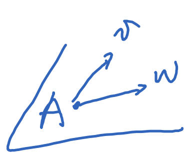
$$\left.\begin{array}{l} A(x_0,y_0,z_0) \\ n=(a,b,c) \\ M(x,y,z) \end{array}\right\} \quad \begin{aligned} \vec{AM}\cdot n &= (x-x_0,\, y-y_0,\, z-z_0)\cdot(a,b,c) \\ &= a(x-x_0)+b(y-y_0)+c(z-z_0) \end{aligned}$$

Conclusion

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0.$$

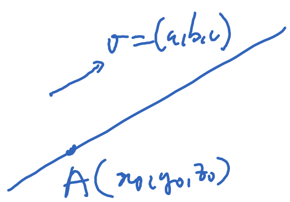
Ex Find the eq. of the plane containing 3 points $A(1,1,1)$, $B(0,1,-1)$, $C(2,-2,0)$.

* Parametric eq. of a plane: If we know a point the plane passes through and two vectors parallel to the plane then the plane has equation: $\vec{AM} = sv + tw$.



Equation of a line

A line is completely determined if we know a point the line passes through and a direction vector



$$\vec{AM} = tv$$

$$(x - x_0, y - y_0, z - z_0) = t(a,b,c)$$

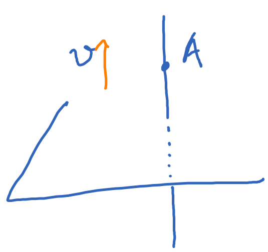
$$\rightsquigarrow \begin{cases} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{cases}$$

This is a parametric eq.

Symmetric eq:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Ex Find eq. of the line passing through $A(1,0,1)$ and perpendicular to the plane $x+y-z=2$.



A normal vector of the plane is also a direction vector of the line: $v = (1,1,-1)$.

Parametric eq. of the line:

$$\begin{cases} x = 1 + t \\ y = t \\ z = 1 - t \end{cases}$$

Quadratic surfaces

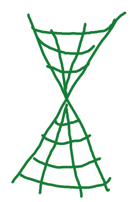
These are surfaces given by an equation of the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz + J = 0$$

Per change of variables, one can reduce this equation to one of the following forms:

(1) $Ax^2 + By^2 + Cz^2 = 1$
- If $A, B, C > 0$: ellipsoid 
- If two > 0, one < 0: hyperboloid (one sheet) 
- If one > 0, two < 0: hyperboloid (two sheets) 

(2) $Ax^2 + By^2 = z^2$ — cone



(3) $Ax^2 + By^2 = z$
- If A, B have the same sign: elliptic paraboloid

- If A, B have opposite signs: hyperbolic paraboloid