Example on finding directional derivatives

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$$f(x,y) = 2xy^2$$

(a) Find the rate of change of f at $(2,1)$ in the direction of $\langle 3,2\rangle$.

(b) Find the direction at which the rate of change is maximum.

Recall: if $u = \langle a,b\rangle$ is a unit vector then

$$D_u f(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+ha, y_0+hb) - f(x_0,y_0)}{h}.$$

$$u = \frac{\langle 3,2\rangle}{\sqrt{13}} = \left\langle \underbrace{\frac{3}{\sqrt{13}}}_{a}, \underbrace{\frac{2}{\sqrt{13}}}_{b} \right\rangle$$

Alternatively,

$$D_u f(x_0,y_0) = \underbrace{u}_{\langle a,b\rangle} \cdot \underbrace{\nabla f(x_0,y_0)}_{\langle f_x, f_y\rangle} = af_x + bf_y.$$

$$\left.\begin{aligned} f_x &= 2y^2 \rightsquigarrow f_x(2,1) = 2 \\ f_y &= 4xy \rightsquigarrow f_y(2,1) = 8 \end{aligned}\right\} \quad D_u f(2,1) = \frac{3}{\sqrt{13}} \times 2 + \frac{2}{\sqrt{13}} \times 8 = \frac{22}{\sqrt{13}}$$