Final exam: Some problems for review

1. Let $F = \langle P, Q \rangle$ be a vector field defined on the entire plane. What is the condition of P and Q for F to be conservative?

2. Let $F = \langle P, Q, R \rangle$ be a vector field defined on the entire space. What is the condition of P, Q, R for F to be conservative?

3. Let $F = \langle 3x^2 + 2xy, x^2 + 2y \rangle$. Is F a conservative vector field? If so, what is a potential function of F?

4. If F is a vector field then $\operatorname{div} F$ is also a vector field. True or false?

5. If F is a vector field then $\operatorname{curl} F$ is also a vector field. True or false?

6. If f is a scalar function then $\operatorname{div}(\nabla f) = 0$. True or false?

7. If F is a vector field then $\operatorname{div}(\operatorname{curl} F) = 0$. True or false?

8. If F is a vector field then $\operatorname{curl}(\operatorname{curl} F) = 0$. True or false?

9. If f is a scalar function then $\operatorname{curl}(\nabla f) = 0$. True or false?

10. The spherical coordinates:
$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}$$
What is the range of θ and ϕ?

11. Convert the Cartesian coordinates $(x, y, z) = (-2, 2, 2\sqrt{6})$ into spherical coordinates.

12. Convert the spherical coordinates $(\rho, \theta, \phi) = (2, \pi/3, \pi/2)$ into Cartesian coordinates.

13. Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transform $x = u^2 + uv$, $y = uv^2$.

14. Find the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ of the transform $x = u^2$, $y = v^2$, $z = w^2$.

15. In the spherical coordinates, $dV =$ __________ $d\rho d\phi d\theta$.

16. Find the volume of the solid enclosed by $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$.

17. Evaluate the integral $\int_C (x^2 + y^2)dx + (x^2 - y^2)dy$ where C is the triangle with vertices at $(0, 0)$, $(2, 1)$, $(0, 1)$.