Lecture 10

Thursday, February 11, 2021 4:15 PM

* Prayer
* Spiritual thought
* Answering questions

Partial derivative:

$$f(x,y) = x^2 + xy - y^3$$

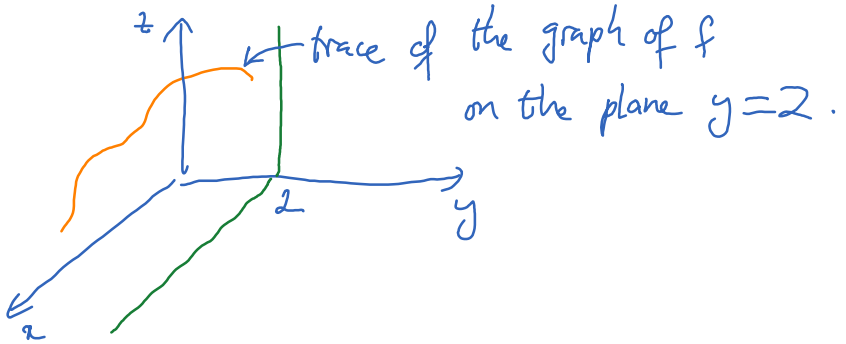
$$\rightsquigarrow f_x(x,y) = 2x + y$$

what about $f_x(1,2)$?

Fix $y=2$, we get a function $g(x) = f(x,2)$.

Then take the derivative of f at $x=1$.

$$f_x(1,2) = g'(1).$$



* Remark about notation:

$$f_x = \frac{\partial f}{\partial x}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

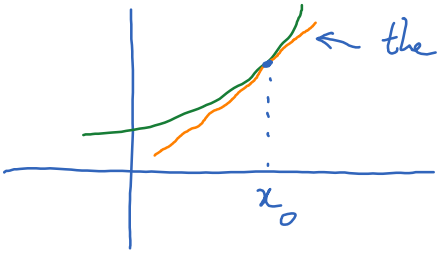
$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

* $f_{xy} = f_{yx}$

$f_{xyyx} \overset{?}{=} f_{yxxy}$

Linear approximation

Recall: $f(x)$



← the line that best approximates f near x_0.

That line (the tangent) passes through $(x_0, f(x_0))$ and direction vector $\langle 1, f'(x_0) \rangle$.

$$\begin{cases} x = x_0 + t \\ y = f(x_0) + f'(x_0)\, t \end{cases} \rightsquigarrow y = f(x_0) + f'(x_0)(x - x_0).$$

What is the purpose of this approximation?

- Linearizing the problem.
- Approximate the error.

$$\Delta f \approx f'(x)\Delta x$$

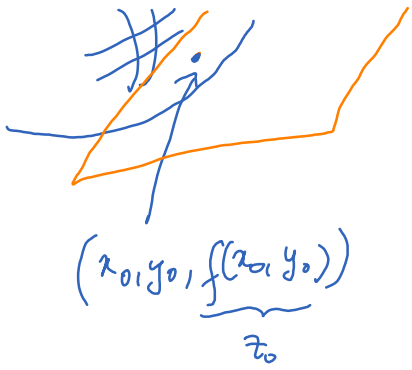
Ex

Use machine to make a pizza of radius R. If there is an error in R, say $\varepsilon(R)$, what is the error in the area of the pizza?

$$S = \pi R^2$$

$$\Delta S \approx 2\pi R \Delta R \lesssim 2\pi R \varepsilon.$$

Now consider $f(x,y)$



$$f(x_0+h, y_0+k)$$

$$\approx f(x_0, y_0+k) + h f_x(x_0, y_0+k)$$

$$\approx f_y(x_0, y_0)\,k + f(x_0, y_0) + h\left(f_x(x_0, y_0) + k f_{xy}(x_0, y_0)\right)$$

$$\approx z_0 + f_x(x_0, y_0)\,h + f_y(x_0, y_0)\,k.$$

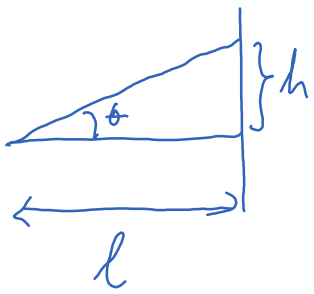
Tangent plane: $z = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + z_0$

Ex: Find the equation of the tangent plane to the ellipsoid

$$x^2 + 2y^2 + 2z^2 = 2$$

at point $\left(1, \frac{1}{2}, \frac{1}{2}\right)$.

Ex:



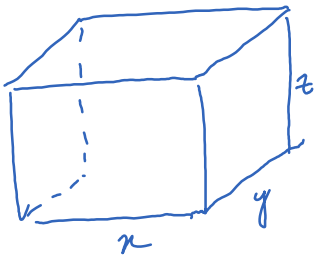
Shooting machine:

$$h = h(\theta, l) = l \tan\theta.$$

If $\theta = \frac{\pi}{4} + \varepsilon_1$ and $l = 20 + \varepsilon_2$

then what is Δh approximately?

Ex



$$V = xyz$$

What is ΔV (approximately)?

Chain rule

$$z = f(x,y) = f(x(t), y(t))$$

Ex

$$z = f(x,y) = xy$$

$$x = t^2$$

$$y = t$$

what is $\frac{dz}{dt}$?

Recall: $[f(g(x))]' = f'(g(x))\, g'(x)$

der. of the "outer" function

der. of the "inner" function

$$\Delta f(g) \approx f'(g)\,\Delta g \approx f'(g)\, g' \Delta x.$$