Lecture 13

Thursday, February 25, 2021 4:15 PM

* Prayer
* Spiritual thought
* Answering questions ....

Extrema of a function:

$$f(x,y) = f(x_0,y_0) + \underbrace{f_x(x_0,y_0)}_{0}\Delta x + \underbrace{f_y(x_0,y_0)}_{0}\Delta y$$

$$+ \frac{1}{2} f_{xx}(x_0,y_0)\Delta x^2 + f_{xy}(x_0,y_0)\Delta x\Delta y$$

$$+ \frac{1}{2} f_{yy}(x_0,y_0)\Delta y^2 + \dots$$

$$r = \frac{\Delta x}{\Delta y} \rightsquigarrow \frac{1}{2} f_{xx} r^2 + f_{xy} r + \frac{1}{2} f_{yy} \quad \begin{matrix} > 0 \\ < 0 \end{matrix}$$

If $D > 0$ and $f_{xx} < 0$: local max

If $D > 0$ and $f_{xx} > 0$: local min

If $D < 0$: saddle point.

If $D = 0$: inconclusive.

Ex:

$$f(x,y) = 3x^2 - x^3 + 2xy + y^2$$

Find local min, local max, abs. min, abs. max.

$$\begin{cases} f_x = 6x - 3x^2 + 2y = 0 \\ f_y = 2x + 2y = 0 \end{cases} \rightsquigarrow \begin{cases} x=0, y=0 \\ x = \frac{4}{3}, y = -\frac{4}{3}. \end{cases}$$

Use second derivative test:

$(0,0)$ is a local min,

$\left(\frac{4}{3}, -\frac{4}{3}\right)$ is a saddle point.

How to find abs. min, abs. max?

Find absolute extrema of a function.

$$f(x,y,\dots) \to \text{max/min on } D$$

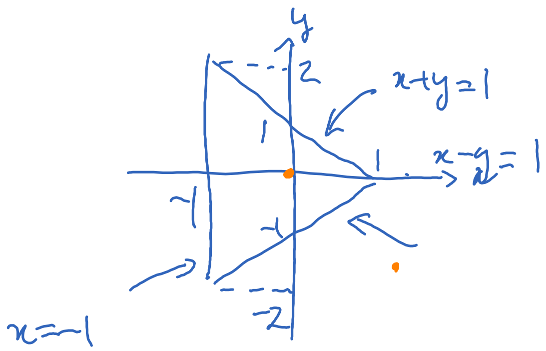
Rule of thumb:

[If max/min exists, it must occur on the boundary of D
or at a critical point inside D.]

* On $\mathbb{R}^2$: no boundary.

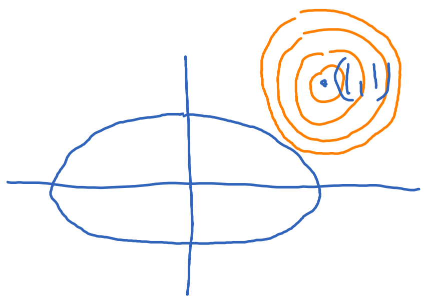
If min/max exists, it must occur at a critical point.

* On the triangle:



Check f on each edge.

* Extrema under constraints



Find the point on the ellipse that is closest to the origin.

$$\begin{cases} (x-1)^2+(y-1)^2 \longrightarrow \min \\ x^2+4y^2=1 \end{cases}$$

More general problem:

$$\begin{cases} f(x,y,\dots) \to \min/\max \\ g(x,y,\dots) = k \quad (\text{fixed}) \\ (x,y,\dots) \in D \end{cases}$$

min/max occurs either on the boundary of D or where

$(x, y, \dots)$ solves this system

$$\begin{cases} \nabla f(x, y, \dots) = \lambda \nabla g(x, y, \dots) \\ g(x, y, \dots) = k \end{cases}$$