Lecture 2

Thursday, January 14, 2021 4:29 PM

* Prayer

* Spiritual thoughts:

  - The joy of learning is part of our experience in mortality (Men are that they may have joy.

  - Record your experience when you realize that what you are learning confirms a Gospel principle.

* Answering questions ...

Dot product:

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Length:

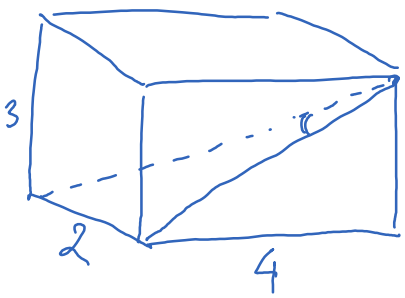
$$|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Angle:

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

(Coming from the Law of Cosine of Triangle)

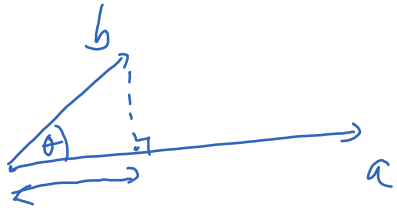
Ex



What is angle θ?

Direction angles, cosines.

Projection:



Scalar projection:

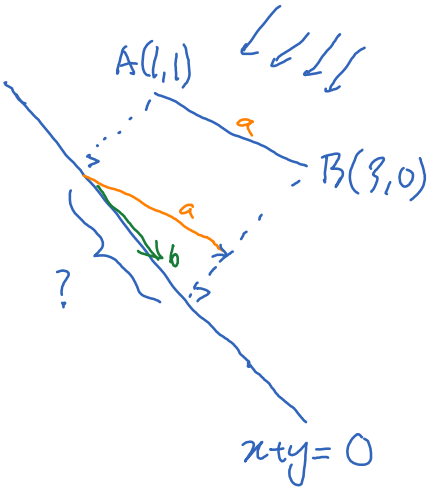
$$\text{comp}_b a = |b|\cos\theta$$

$$= |b|\frac{a\cdot b}{|a||b|} = \frac{a\cdot b}{|a|}$$

Vector projection:

$$\text{proj}_b a = \frac{a}{|a|}\,\text{comp}_a a = \frac{a\cdot b}{|a|^2}\,a$$

Ex:



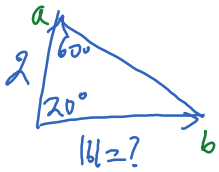
what is the length of the shadow of AB?

$$a = \overrightarrow{AB} = \langle 2,-1\rangle$$

$$b = \langle 1,-1\rangle$$

$$\text{Comp}_b a = \frac{a\cdot b}{|b|} = \frac{2(1)+(-1)(-1)}{\sqrt{1^2+(-1)^2}} = \frac{3}{\sqrt{2}}$$

Ex



$$(a-b)\cdot a = |a||a-b|\cos 60^\circ = \frac{1}{2}|a||a-b|$$

$$a\cdot b = |a||b|\cos 20^\circ$$

$$|a|^2 - a\cdot b = \frac{1}{2}|a||a-b|$$

($|a|^2 = 4$)

$$(a-b)(a-b) = |a|^2 + |b|^2 - 2|a||b|\cos 20^\circ$$

$$= 4 + |b|^2 - 4|b|\cos 20^\circ$$

$$\rightsquigarrow \quad 4 - \frac{|a|}{2}|b|\cos 20^\circ = \frac{1}{2}\underbrace{|a|}_{2}|a-b|$$

Cross product:

$$a \times b = \langle a_1, a_2, a_3 \rangle \times \langle b_1, b_2, b_3 \rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$a \times b \perp a$

$a \times b \perp b$

Question: $(a \times b) \times c \stackrel{?}{=} a \times (b \times c)$

Parallelogram $|a \times b| =$ area.