

Lecture 25

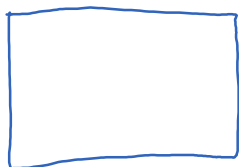
Thursday, April 8, 2021 3:49 PM

- * Prayer
 - * Spiritual thought
 - * Answering questions....
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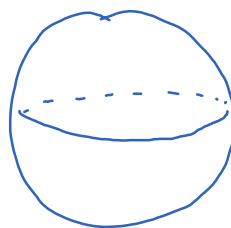
Surface area:

$$S: \begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases}$$

$$\text{Area of } S = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA$$



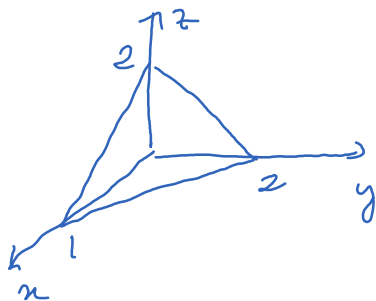
D



S

[find the surface area
of the earth from the map]

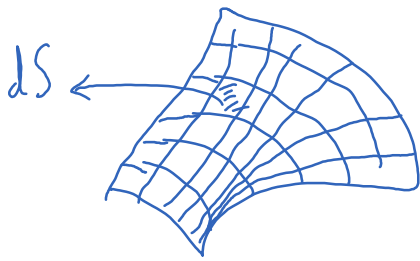
Ex



$$2x + y + z = 2$$

$$S: \begin{cases} x = x \\ y = y \\ z = 2 - 2x - y \end{cases}$$

Surface integral

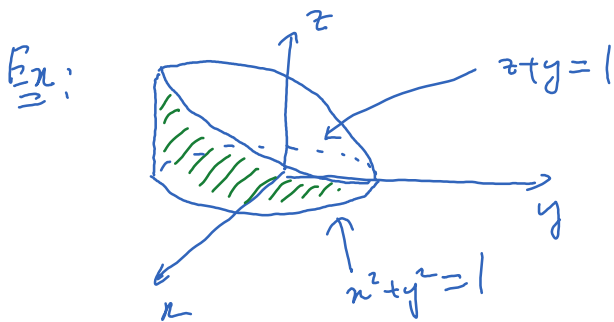


line integral $\left\{ \begin{array}{l} \text{of a scalar func: area of fence,} \\ \text{weight of a wire} \\ \text{of a vector field: work,} \\ \text{Circulation} \end{array} \right.$

surface integral $\left\{ \begin{array}{l} \text{of a scalar function: weight of a surface} \\ \text{of a vector field: flux} \end{array} \right.$

f : mass density

$$\text{Total mass} \approx \sum f \Delta S = \iint_S f dS = \iint_D f(\mathbf{r}) |\mathbf{r}_u \times \mathbf{r}_v| dA.$$



$f(x, y, z) = xy$ ← mass density (per unit area)

$$\iint_S f dS = ?$$

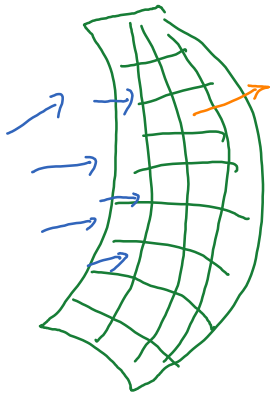
$$S: \begin{cases} x = \cos \theta \\ y = \sin \theta \\ z = z \end{cases} \quad \begin{array}{l} 0 \leq \theta \leq 2\pi \\ 0 \leq z \leq 1 - \sin \theta \end{array}$$

$\underbrace{\hspace{10em}}_D$

$$\left. \begin{array}{l} \mathbf{r}_\theta = \langle x_\theta, y_\theta, z_\theta \rangle = \langle -\sin \theta, \cos \theta, 0 \rangle \\ \mathbf{r}_z = \langle x_z, y_z, z_z \rangle = \langle 0, 0, 1 \rangle \end{array} \right\} \mathbf{r}_\theta \times \mathbf{r}_z = \langle \cos \theta, \sin \theta, 0 \rangle$$

$$\begin{aligned} \iint_D f(r) \underbrace{|\mathbf{r}_u \times \mathbf{r}_v|}_{1} dA &= \iint_S xy \, dA \\ &= \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, dz \, d\theta \\ &= \dots \end{aligned}$$

Surface integral of a vector field



Flux across S is $\iint_S \vec{F} \cdot \vec{n} \, dS$

$$d\vec{S} = \vec{n} \, dS$$

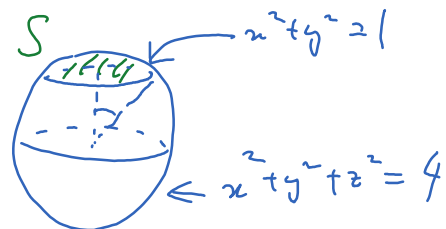
$$\iint_S \vec{F} \cdot d\vec{S}$$

How to compute:

$$\vec{n} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$$

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n} \, dS = \iint_D \vec{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, dA$$

Ex: $\vec{F} = \langle xy, z, z \rangle$



$$S: \begin{cases} x = \sin \phi \cos \theta \\ y = \sin \phi \sin \theta \\ z = \cos \phi \end{cases} \quad \begin{array}{l} 0 \leq \phi \leq \frac{\pi}{6} \\ 0 \leq \theta \leq 2\pi \end{array}$$

D

$$\begin{aligned} r_\phi &= \langle x_\phi, y_\phi, z_\phi \rangle \\ &= \langle \cos \phi \cos \theta, \cos \phi \sin \theta, -\sin \phi \rangle \end{aligned}$$

$$r_\theta = \langle -\sin \phi \sin \theta, \sin \phi \cos \theta, 0 \rangle$$

$$r_\phi \times r_\theta = \langle \sin^2 \phi \cos \theta, \sin^2 \phi \sin \theta, \cos \phi \sin \phi \rangle$$

$$r_\phi \times r_\theta = \langle \sin \phi \cos \phi \dots \rangle$$

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \dots$$

$\vec{F} \cdot \vec{n}$

