Lecture 3

Tuesday, January 19, 2021 2:10 PM

* Prayer

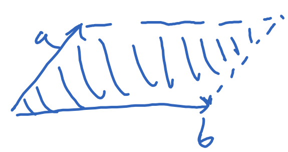
* Spiritual thought: "I will bless all those who labor in my vinyard with a mighty blessing..." (D&C 21:9)

* Answering questions...

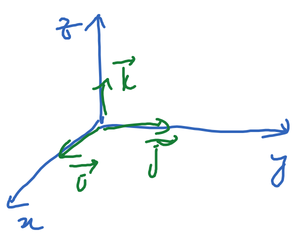
Cross product

$a \times b \perp a, b$, direction determined by right hand rule.

$|a \times b| =$ area of parallelogram



Ex:



$\vec{i} \times \vec{j} = \vec{k}$

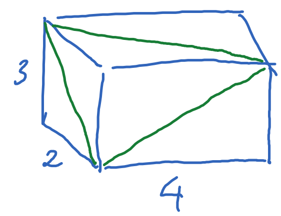
$\vec{j} \times \vec{k} = \vec{i}$

$\vec{k} \times \vec{i} = \vec{j}$

Algebraically, $a \times b = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$

$$\begin{matrix} a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{matrix}$$

Ex



What is the area of the green triangle?

Ex Is it true that

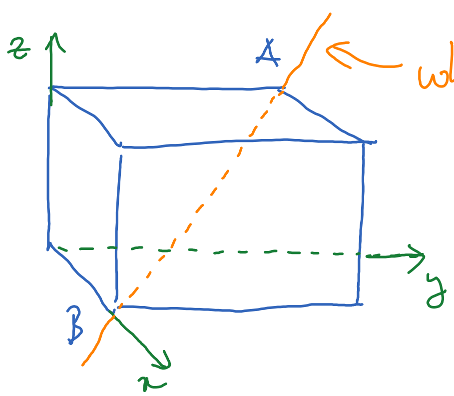
$$a \times b = b \times a \quad ?$$

$$(a \times b) \times c = a \times (b \times c) \quad ?$$

Equation of lines

- Vector equation $\longrightarrow$ $\vec{r} = \vec{r}_0 + t\vec{a}$
- parametric equation $\longrightarrow$ $\begin{cases} x = x_0 + t a_1 \\ y = y_0 + t a_2 \\ z = z_0 + t a_3 \end{cases}$
- symmetric equation $\longrightarrow$ $\dfrac{x - x_0}{a_1} = \dfrac{y - y_0}{a_2} = \dfrac{z - z_0}{a_3}$

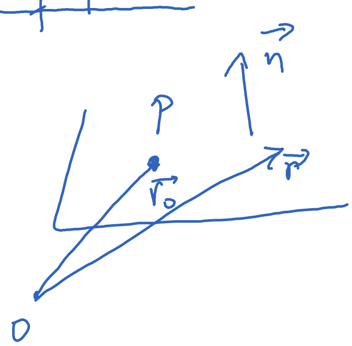
Ex



what is the equation of this line?

What is the equation of the line segment from A to B?

Equation of planes



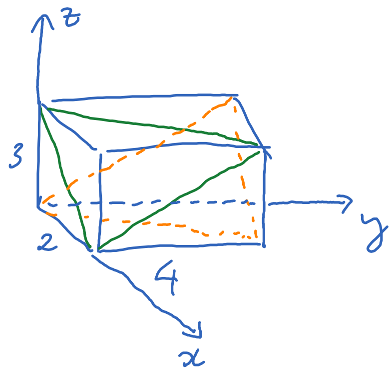
$$\boxed{(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0}$$

vector eq.

Scalar equation $\quad ax + by + cz = ax_0 + by_0 + cz_0$

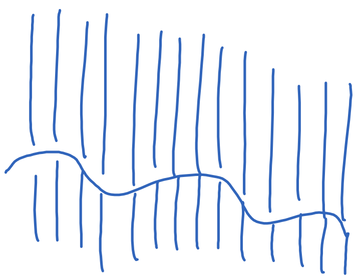
where $\vec{n} = \langle a, b, c \rangle$ and $P(x_0, y_0, z_0)$.

Ex

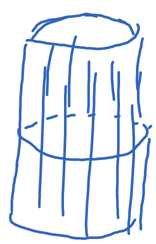


- Find the equation of the green plane.
- Find the equation of the red plane
- Find the equation of the intersection of the two planes.

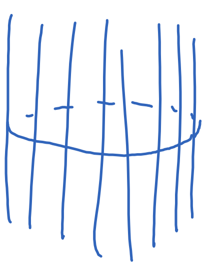
Cylinder surfaces



A cylinder surface is a "wall" built on a curve.



circular cylinder



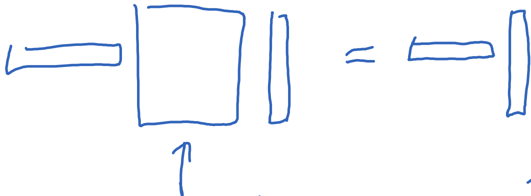
elliptic cylinder

parabolic cylinder.

Quadratic surfaces are of the form $Ax^2+By^2+Cz^2+Dyz+Ezx+Fxy$
$+Gx+Hy+Iz+J=0$

Put $u=\begin{bmatrix}x\\y\\z\end{bmatrix}$.

$$u^TQu = Pu$$



symmetric

diagonalizable

$$v^T\begin{bmatrix}\lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3\end{bmatrix}v = Rv$$

$$\lambda_1x^2+\lambda_2y^2+\lambda_3z^2 = r_1x+r_2y+r_3z$$

$$\lambda_1\left(x-\frac{r_1}{2\lambda_1}\right)^2+\lambda_2\left(y-\frac{r_2}{2\lambda_2}\right)^2+\lambda_3z^2 = \# + r_2z$$

In most cases, the equation can be "reduced" to

$$Ax^2+By^2+Cz^2+J=0$$

$$Ax^2+By^2+Cz+J=0$$

Plotting with Mathematica:

Plot3D, ContourPlot3D