Lecture 4

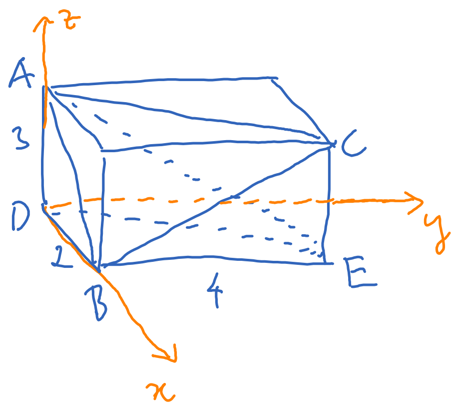
Thursday, January 21, 2021 3:54 PM

* Prayer
* Spiritual thought : Elder Bednar's BYU Devotional talk ...

  During hard times, we should count blessings, miracles and acknowledge the goodness of God.

* Answering questions ....

Another example on planes and lines :



Find the equation of the planes ABC, ADE and the intersection between them.

A(0,0,3)

B(2,0,0)

C(2,4,3)

D(0,0,0)

E(2,4,0)

$$\left.\begin{array}{l}\vec{AB} = \langle 2,0,-3\rangle \\ \vec{AC} = \langle 2,4,0\rangle\end{array}\right\}$$ normal vector of the plane ABC is

$$\vec{n}_1 = \vec{AB} \times \vec{AC} = \langle 12, -6, 8\rangle$$

Eq. of the plane ABC is

$$\vec{n}_1 \cdot \left(\langle x,y,z\rangle - \langle 0,0,3\rangle\right) = 0$$

$$\rightsquigarrow 12(x-0) + (-6)(y-0) + 8(z-3) = 0$$

$$\rightsquigarrow 12x - 6y + 8z = 24 \rightsquigarrow \boxed{6x - 3y + 4z = 12}$$

$$\left.\begin{aligned}\vec{DA} &= \langle 0,0,3\rangle \\ \vec{DB} &= \langle 2,4,0\rangle\end{aligned}\right\}$$ normal vector of the plane ADE is

$$\vec{n}_2 = \vec{DA} \times \vec{DE} = \langle -12, 6, 0\rangle$$

Equation of the plane ADE:

$$\vec{n}_2 \cdot (\langle x,y,z\rangle - \underbrace{\langle 0,0,0\rangle}_{D}) = 0$$

$$\langle -12, 6, 0\rangle \cdot \langle x,y,z\rangle = 0$$

$$\rightsquigarrow -12x + 6y + 0z = 0$$

$$\rightsquigarrow \boxed{-2x + y = 0}$$

Intersection:

$$\begin{cases} 6x - 3y + 4z = 12 \rightsquigarrow n_1 = \langle 6, -3, 4\rangle \\ -2x + y = 0 \rightsquigarrow n_2 = \langle -2, 1, 0\rangle \end{cases}$$

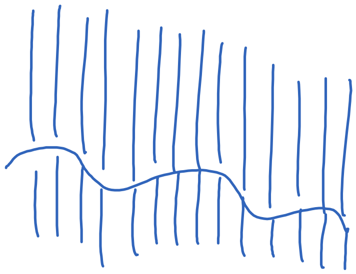
Intersection is a line with direction vector

$$a = n_1 \times n_2 = \langle -4, -8, 12\rangle \rightsquigarrow \langle -1, -2, 3\rangle$$

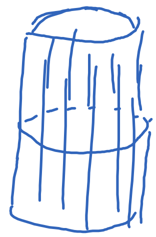
$A(0,0,3)$ lies on the line. The parametric equation of the line is

$$\begin{cases} x = 0 + (-1)t \\ y = 0 + (-2)t \\ z = 3 + 3t \end{cases}$$

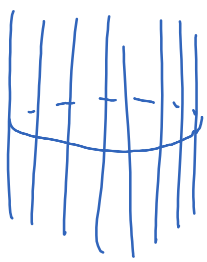
Cylinder surfaces



A cylinder surface is a "wall" built on a curve.



circular cylinder



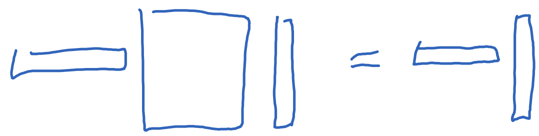
elliptic cylinder

parabolic cylinder.

<u>Quadratic surfaces</u> are of the form $Ax^2+By^2+Cz^2+Dyz+Ezx+Fxy+Gx+Hy+Iz+J=0$

Put $u=\begin{bmatrix}x\\y\\z\end{bmatrix}$.

$$u^TQu = Pu$$



symmetric

diagonalizable

$$v^T\begin{bmatrix}\lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3\end{bmatrix}v = Rv$$

$$\lambda_1x^2+\lambda_2y^2+\lambda_3z^2 = r_1x+r_2y+r_3z$$

$$\lambda_1\left(x-\frac{r_1}{2\lambda_1}\right)^2+\lambda_2\left(y-\frac{r_2}{2\lambda_2}\right)^2+\lambda_3z^2 = \# + r_3z$$

In most cases, the equation can be "reduced" to

$$Ax^2+By^2+Cz^2+J=0$$

$$Ax^2+By^2+Cz+J=0$$

Plotting with Mathematica:

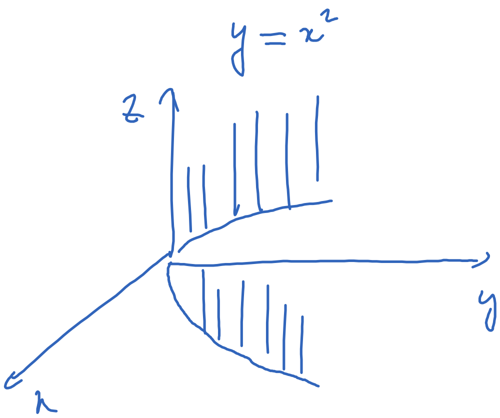
Plot3D, ContourPlot3D

Describe traces on planes....

Vector functions

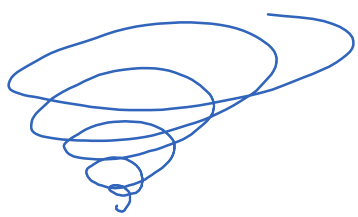
$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ (sometimes the arrow on $\vec{r}$ is dropped.)

Ex $\vec{r}(t) = \langle t, t^2, t^3 \rangle$



The curve is contained in the cylindrical surface $y = x^2$.
On this surface, $z = x^3$.
That's why the curve is called a "twisted cubic".

Ex:
$\vec{r}(t) = \langle t\cos t, t\sin t, t \rangle$



Derivative

$$r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$$