Lecture 7

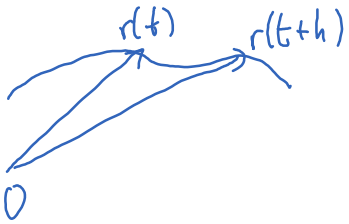
Tuesday, February 2, 2021 11:31 AM

* Prayer

* Spiritual thought:

* Answering questions ....

$r(t)$ ... position at time t

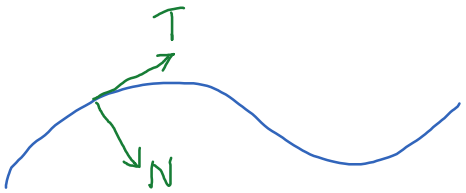


$$v(t) = r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$$

is the velocity.

Speed: $V(t) = |v(t)|$

Acceleration: $a(t) = r''(t)$



normal vector

We can write $a(t)$ as

$$a = \underbrace{V'(t)T}_{\text{tangential component}} + \underbrace{\kappa(t)V(t)^2 N}_{\text{normal component}}.$$

Ex:

$$r(t) = \langle t^2 - 1,\ t^3 - 2t,\ 0 \rangle$$

Find velocity, acceleration, speed, normal component of the acceleration at the position $(0, -3, 0)$

$r'(t) = \langle 2t, 3t^2 - 2, 0 \rangle$

$V(t)^2 = 4t^2 + (3t^2-2)^2 = 9t^4 - 8t^2 + 4 = \left(3t^2 - \frac{4}{3}\right)^2 + \frac{20}{9}$

Minimum speed is $\sqrt{\frac{20}{9}}$, at $t = \frac{2}{3}$.

$r''(t) = \langle 2, 6t, 0 \rangle$

$r'(t) \times r''(t) = \langle 0, 0, 12t^2 - 2(3t^2-2) \rangle$

$= \langle 0, 0, 4 + 6t^2 \rangle$

$$\kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{4+6t^2}{\left[\left(3t^2 - \frac{4}{3}\right)^2 + \frac{20}{9}\right]^{3/2}}$$

* Problem 3 & 9, page 928.

9) The helix $r_1(t) = \langle \cos t, \sin t, t \rangle$

and the curve $r_2(t) = \langle 1+t, t^2, t^3 \rangle$

intersect at point $(1,0,0)$. Find the angle of intersection.

3) Find a vector function that represents the curve of intersection of $x^2 + y^2 = 16$ and $x + z = 16$.

Save for midterm review

* Function of several variables

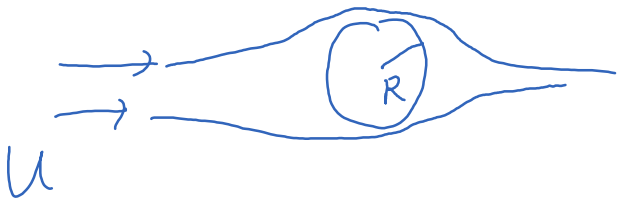
description by { table, formula, graph } → suitable for functions with more than two variables.

Ex: Body mass index $BMI = \frac{m}{h^2}$ $\left(\frac{kg}{m^2}\right)$

m \ h	1.5	1.51	1.52	...

$18.5 < BMI < 25$: OK

Ex



Drag force : depending on

U, R, ρ, ν

- U: velocity of the flow
- ρ: density of fluid
- ν: viscosity

→ experiment.

* Graph of a two-variable function :

is a surface

One can talk about { domain, range, level set

Ex $$f(x,y) = \frac{\sqrt{x+y}}{\sqrt{x-y}} \log(y^2 - x)$$

One can use Mathematica to plot the domain (RegionPlot) and level set (ContourPlot or ContourPlot3D).

Ex: $f(x,y) = x^2 + 4y^2 + 1$ $\longrightarrow$ level set is an ellipsoid

$f(x,y) = \sqrt{x^2+4y^2}$

$f(x,y) = \ln(y - x^2)$

$f(x,y,z) = x^2 + y^2 - z^2$

Limit

(x_0, y_0) $\qquad \lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$

Ex $$f(x,y) = \frac{xy}{x^2+y^2}$$

$$f(x,y) = \frac{x^2 y}{x^2+y^2}$$