Tangent plane of ellipsoid

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Find the tangent plane to the ellipsoid

$$x^2+2y^2+2z^2=2$$

at point $\left(1,\frac{1}{2},\frac{1}{2}\right)$.

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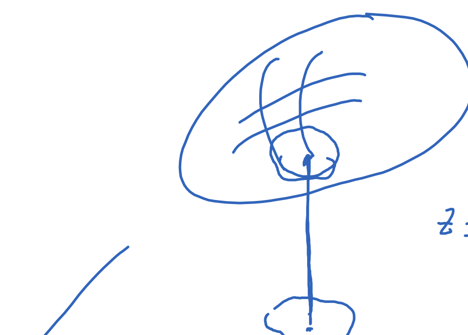
* Recall: the tangent plane of the graph of function $f(x,y)$ at point $(x_0,y_0,f(x_0,y_0))$ has the equation

$$z=f(x_0,y_0)+f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0).$$

$$z^2=\frac{2-x^2-2y^2}{2}$$

$$z=\underbrace{\sqrt{\frac{2-x^2-2y^2}{2}}}_{f(x,y)}$$



$\left(1,\frac{1}{2}\right)$

$z=\frac{1}{2}>0$

$x_0=1,\ y_0=\frac{1}{2}$

Eq. of the tgent plane is $z=f\left(1,\frac{1}{2}\right)+f_x\left(1,\frac{1}{2}\right)(x-1)$
$+f_y\left(1,\frac{1}{2}\right)\left(y-\frac{1}{2}\right)$

$$f(x,y) = \sqrt{\frac{2-x^2-2y^2}{2}} = \left(\frac{2-x^2-2y^2}{2}\right)^{1/2}$$

$$f_x = \frac{1}{2}(-x)\left(\frac{2-x^2-2y^2}{2}\right)^{-1/2}$$

$$f_x\left(1,\tfrac{1}{2}\right) = -1$$

$$f_y\left(1,\tfrac{1}{2}\right) = -1$$

$$z = \frac{1}{2} + (-1)(x-1) + (-1)\left(y-\frac{1}{2}\right)$$

$$\boxed{x+y+z = 2}$$