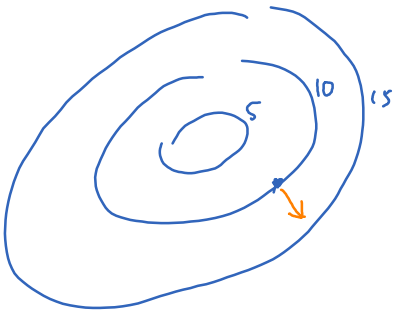
Extrema of a function

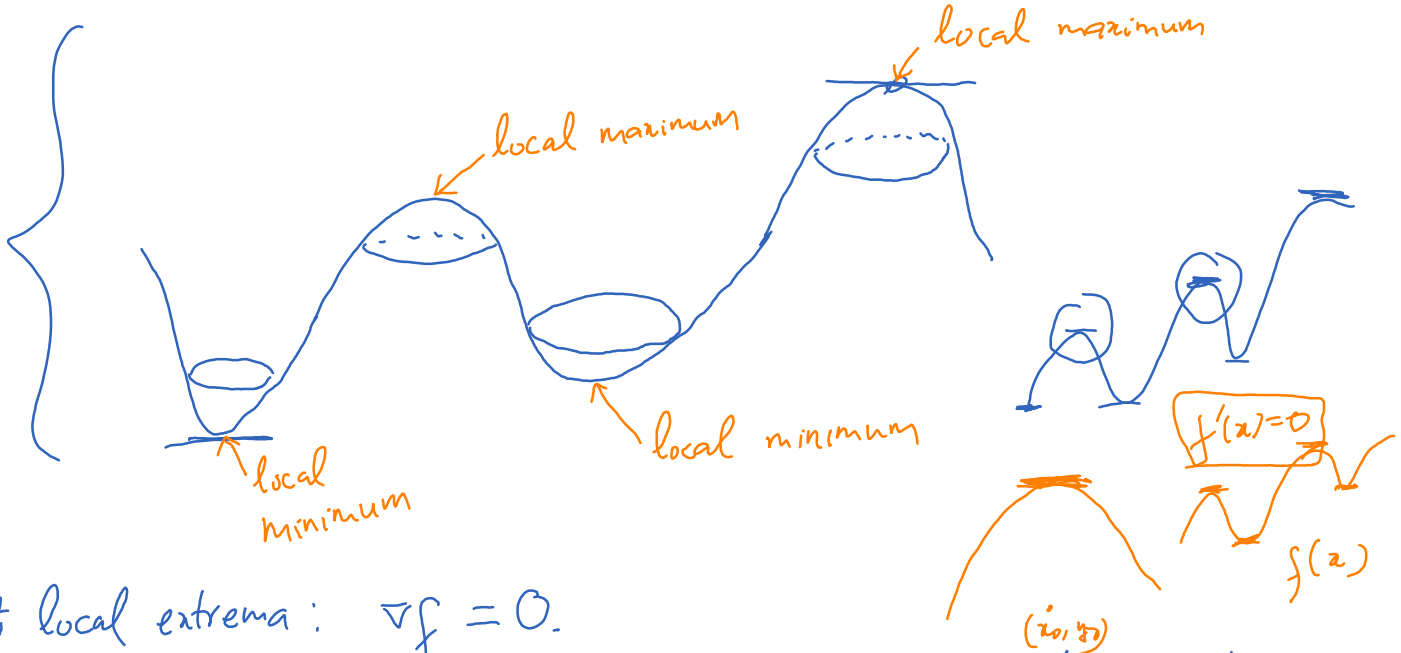
Saturday, February 20, 2021 5:03 PM

Recall: the gradient vector at a point is perpendicular to the level set passing through that point.

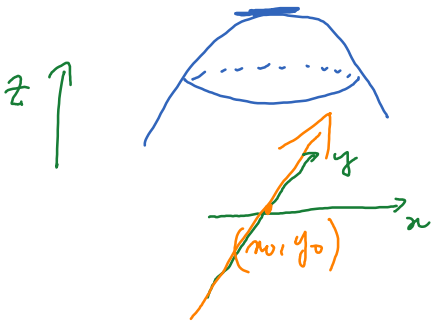


Question: At the point where f attains maximum, what is the gradient of f?

Local extrema:



At local extrema: $\nabla f = 0$.



Rate of change in the x-direction:

$$f_x(x_0, y_0) = 0.$$

Rate of change in the y-direction:

$$f_y(x_0, y_0) = 0.$$

$\Rightarrow \nabla f = 0$

A critical point is where $\nabla f = 0$.

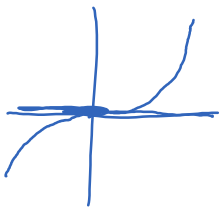
$\nabla f(x_0, y_0) = 0$

Critical point $\begin{cases} \text{local minimum} \\ \text{local maximum} \\ \text{saddle point} \end{cases}$

$f'(x_0) = 0$

Ex: $f(x,y) = x^2 + xy + y^2 + y$.

$f(x) = x^3$



$f'(0) = 0$

$f_x = 2x + y$

$f_y = x + 2y + 1$

Solve for critical point:

$$\begin{cases} 2x + y = 0 \\ x + 2y + 1 = 0 \end{cases} \rightsquigarrow \begin{cases} x = 1/3 \\ y = -2/3 \end{cases}$$

How do we know if $\left(\frac{1}{3}, -\frac{2}{3}\right)$ is where f attains local maximum or minimum?

Ex $f(x,y) = (x-y)(1-xy)$

Find the critical points of f.