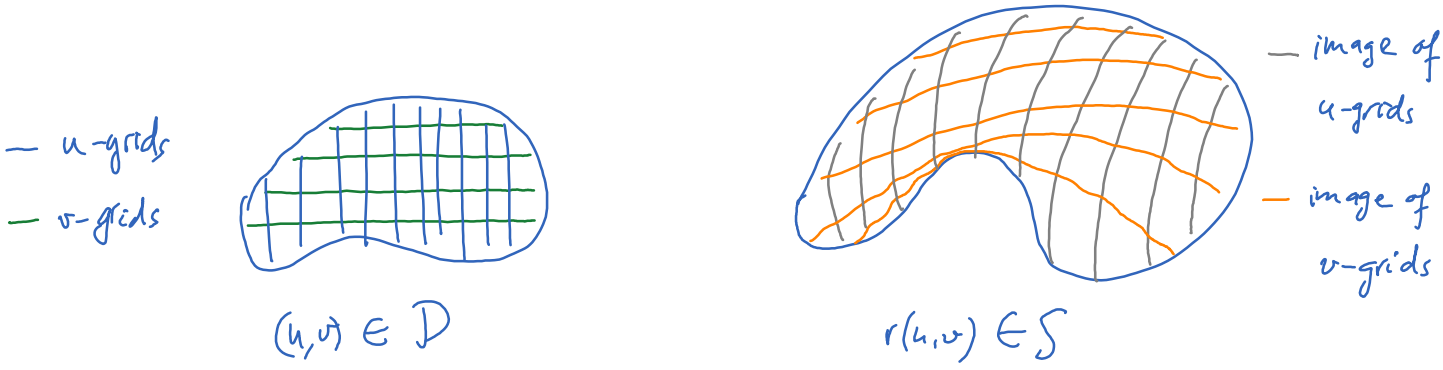
Finding surface area

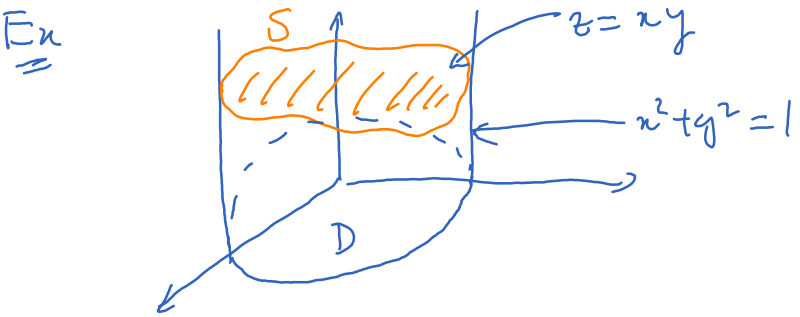
Tuesday, April 6, 2021 7:19 AM



$(u,v) \in D$ $\qquad$ $r(u,v) \in S$

$$\text{Area of } S = \iint_D |r_u \times r_v| \, dA$$

Ex



Parametrize the surface:

$$r: \begin{cases} x = x \\ y = y \\ z = xy \end{cases} \qquad (x,y) \in D - \text{unit disc}$$

$r = r(x,y) = \langle x, y, xy \rangle$ (x: u, y: v)

$r_x = \langle 1, 0, y \rangle$

$r_y = \langle 0, 1, x \rangle$

$$r_x \times r_y = \begin{vmatrix} 1 & 0 & y \\ 0 & 1 & x \end{vmatrix} \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \langle -y, -x, 1 \rangle$$

$$\text{Area of } S = \iint_D |r_x \times r_y| \, dA = \iint_D \sqrt{x^2+y^2+1} \, dA$$

$$= \int_0^1 \int_0^{2\pi} \sqrt{\underbrace{r^2+1}_{u}} \, r \, d\theta \, dr$$

$$= \int_1^2 \int_0^{2\pi} \sqrt{u} \, \frac{u}{2} \, d\theta \, du = \cdots$$