Lecture 10

Wednesday, February 3, 2021 2:31 PM

* Prayer
* Spiritual thought
* Answering questions ---

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* Function of several variables

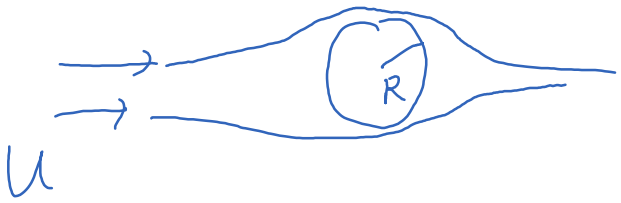
description by: table, formula, graph → suitable for functions with more than two variables.

Ex: Body mass index $BMI = \frac{m}{h^2}$ $\left(\frac{kg}{m^2}\right)$

m \ h	1.5	1.51	1.52	...

$18.5 < BMI < 25$: OK

Ex



Drag force : depending on

U, R, ρ, ν

- U: velocity of the flow
- ρ: density of fluid
- ν: viscosity

$\longrightarrow$ experiment.

* Graph of a two-variable function : is a surface

One can talk about { domain, range, level set

Ex

$$f(x,y) = \frac{\sqrt{x+y}}{\sqrt{x-y}} \log(y^2 - x)$$

One can use Mathematica to plot the domain (RegionPlot) and level set (ContourPlot or ContourPlot3D).

Ex: $f(x,y) = x^2 + 4y^2 + 1$ $\longrightarrow$ level set is an ellipsoid

$f(x,y) = \sqrt{x^2+4y^2}$

$f(x,y) = \ln(y - x^2)$

$f(x,y,z) = x^2 + y^2 - z^2$

Limit

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$ ((x_0,y_0))

Ex $f(x,y) = \dfrac{xy}{x^2+y^2}$

$f(x,y) = \dfrac{x^2y}{x^2+y^2}$

Mathematica:

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Limit[ xy/(x^2+y^2) , {x,y} -> {0,0}]
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Note:

$$\lim_{(x,y)\to(a,b)} f(x,y) \neq \lim_{x\to a}\lim_{y\to b} f(x,y)$$

Can you find an example?

$f(x,y) = \frac{x}{y}$

$\lim_{y\to 0} \lim_{x\to 0} f(x,y) = 0$

$\lim_{x\to 0} \lim_{y\to 0} f(x,y)$ DNE

$f(x,y) = \frac{x}{x+y}$

$\lim_{y\to 0} \lim_{x\to 0} f(x,y) = 0$

$\lim_{x\to 0} \lim_{y\to 0} f(x,y) = 1.$