Lecture 11

Friday, February 5, 2021 4:11 PM

* Prayer

* Spiritual thought: God answers the prayer of faith.

$\longrightarrow$ pray for understanding what you learn.

* Answering questions ...

Limit

(x_0, y_0)

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

Ex

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$\rightsquigarrow$ To show that the limit doesn't exist, we only need to show that the limits along two different paths are different.

$$f(x,y) = \frac{x^2 y}{x^2+y^2}$$

$\rightsquigarrow$ Compare the order of the numerator and the denominator.

[Use Squeeze theorem]

Mathematica:

Limit[$xy/(x^2+y^2)$, $\{x,y\} \to \{0,0\}$]

Note:

$$\lim_{(x,y)\to(a,b)} f(x,y) \neq \lim_{x\to a} \lim_{y\to b} f(x,y)$$

Can you find an example?

$$f(x,y) = \frac{x}{y} \qquad \lim_{y\to 0} \lim_{x\to 0} f(x,y) = 0$$

$$\lim_{x\to 0} \lim_{y\to 0} f(x,y) \quad \text{DNE}$$

$$f(x,y) = \frac{x}{x+y} \qquad \lim_{y\to 0} \lim_{x\to 0} f(x,y) = 0$$

$$\lim_{x\to 0} \lim_{y\to 0} f(x,y) = 1.$$

Continuity:

A function $f: D \subset \mathbb{R}^n \to \mathbb{R}$ is said to be continuous at $A \in D$ if $\lim_{x\to A} f(x) = f(A)$.

Polynomials are continuous everywhere.

Rational functions are continuous where the denominators are nonzero.

Sum of two continuous functions is a continuous function.

Composition of two continuous function is a continuous function.

Ex:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+2y^2}{3x^2+y^2}$$

$$\lim_{(x,y)\to(1,2)} \frac{2x-y}{4x^2-y^2}$$

$$\lim_{(x,y)\to(1,2)} \frac{2x+y}{4x^2-y^2}$$

$$\lim_{(x,y)\to(0,0)} xy\sin\left(\frac{1}{x^2+y^2}\right)$$

$$\lim_{(x,y)\to(0,0)} (xy+1)\sin\left(\frac{1}{x^2+y^2}\right)$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+x^2y}{x^2+y^2}$$