# Lecture 12

Monday, February 8, 2021 4:20 PM

The following exercises are taken from the textbook (the review problems of Chapter 12 and 14).

1. If $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$, then $\mathbf{u} \cdot \mathbf{v} = \langle u_1 v_1, u_2 v_2 \rangle$.

   Answer

2. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $|\mathbf{u} + \mathbf{v}| = |\mathbf{u}| + |\mathbf{v}|$.

3. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $|\mathbf{u} \cdot \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|$.

   Answer

4. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|$.

5. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$.

   Answer

6. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

7. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $|\mathbf{u} \times \mathbf{v}| = |\mathbf{v} \times \mathbf{u}|$.

   Answer

8. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$ and any scalar $k$,

$$k(\mathbf{u} \cdot \mathbf{v}) = (k\mathbf{u}) \cdot \mathbf{v}$$

9. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$ and any scalar $k$,

$$k(\mathbf{u} \times \mathbf{v}) = (k\mathbf{u}) \times \mathbf{v}$$

Answer

10. For any vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $V_3$,

$$(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$$

11. For any vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $V_3$,

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$$

12. For any vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $V_3$,

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$$

13. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.

Answer

14. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $V_3$, $(\mathbf{u} + \mathbf{v}) \times \mathbf{v} = \mathbf{u} \times \mathbf{v}$.

15. The vector $\langle 3, -1, 2 \rangle$ is parallel to the plane

$$6x - 2y + 4z = 1$$

Answer

16. A linear equation $Ax + By + Cz + D = 0$ represents a line in space.

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17. The set of points $\{(x, y, z) \mid x^2 + y^2 = 1\}$ is a circle.

Answer ↓

18. In $\mathbb{R}^3$ the graph of $y = x^2$ is a paraboloid.

19. If $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

Answer ↓

20. If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

21. If $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

Answer ↓

22. If $\mathbf{u}$ and $\mathbf{v}$ are in $V_3$, then $|\mathbf{u} \cdot \mathbf{v}| \leqslant |\mathbf{u}|\,|\mathbf{v}|$.

18. The plane through $(2, 1, 0)$ and parallel to $x + 4y - 3z = 1$

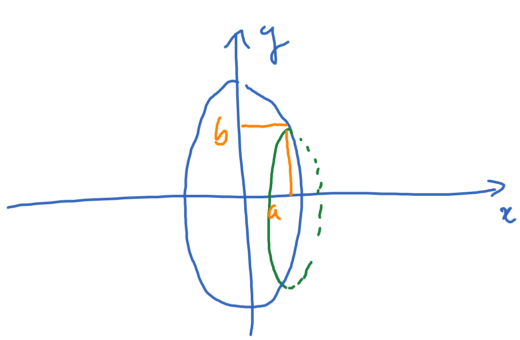
19. The plane through $(3, -1, 1)$, $(4, 0, 2)$, and $(6, 3, 1)$

Answer ↓

20. The plane through $(1, 2, -2)$ that contains the line $x = 2t$, $y = 3 - t$, $z = 1 + 3t$

37. An ellipsoid is created by rotating the ellipse $4x^2 + y^2 = 16$ about the $x$-axis. Find an equation of the ellipsoid.

Solution to Problem 37:




A point $(a, b)$ on the ellipse sweeps a circle centered at $(a, 0)$ in the rotation about the $x$-axis. On this circle,
$$\begin{cases} x = a \\ y^2 + z^2 = b^2 \end{cases}$$

Because $4a^2 + b^2 = 16$, we get $4x^2 + y^2 + z^2 = 16$. This is an equation of the ellipsoid.

1. $f(x, y) = \ln(x + y + 1)$

Answer

2. $f(x, y) = \sqrt{4 - x^2 - y^2} + \sqrt{1 - x^2}$

3–4 Sketch the graph of the function.

3. $f(x, y) = 1 - y^2$

Answer

4. $f(x, y) = x^2 + (y - 2)^2$

5–6 Sketch several level curves of the function.

5. $f(x, y) = \sqrt{4x^2 + y^2}$

Answer

6. $f(x, y) = e^x + y$