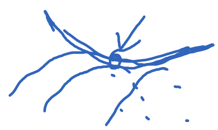
Lecture 18

Monday, February 22, 2021 2:04 PM

* Prayer
* Spiritual thought
* Answering questions.....

- Local extrema
- Absolute extrema
- If f attains local extremum at at point (x_0, y_0) and f is differentiable at (x_0, y_0) then $\nabla f(x_0, y_0) = 0$.

Critical point
- local min
- local max
- saddle point



- How to check if a critical point is where f attains local min/max?

Recall: $$f(x) = f(x_0) + \underbrace{f'(x_0)(x - x_0)}_{=0} + \underbrace{\frac{f''(x_0)}{2}}_{\substack{>0 \\ <0 \\ =0}}(x - x_0)^2 + \cdots$$

$$f(x,y) = f(x_0,y_0) + \underbrace{f_x(x_0,y_0)}_{0}\Delta x + \underbrace{f_y(x_0,y_0)}_{0}\Delta y$$

$$+ \frac{1}{2} f_{xx}(x_0,y_0)\Delta x^2 + f_{xy}(x_0,y_0)\Delta x \Delta y$$

$$+ \frac{1}{2} f_{yy}(x_0,y_0)\Delta y^2 + \ldots .$$

$\frac{\Delta x}{\Delta y} = r$. Look at the sign of

$$\frac{1}{2} f_{xx} r^2 + f_{xy} r + \frac{1}{2} f_{yy} .$$

$$D = f_{xx} f_{yy} - f_{xy}^2$$

If $D > 0$ and $f_{xx} > 0$: local min

If $D > 0$ and $f_{xx} < 0$: local max

If $D < 0$: saddle point.

If $D = 0$: inconclusive.

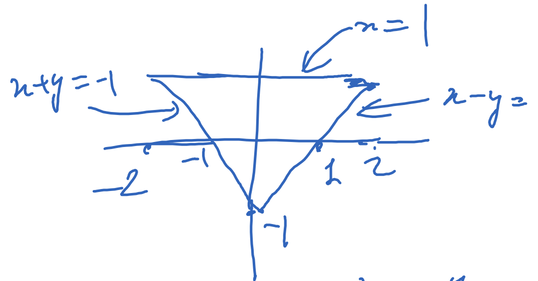
Ex Find critical points at $f(x,y) = 2 - x^4 + 2x^2 - y^2$.

Where does f attain local min / local max ?

Find abs. min / abs. max of a function on a region D

$$f(x,y) = \cancel{x} + 3x^2 - x^3 + 2xy + y^2 \quad \text{on } D = [-1,1] \times [-1,1] \text{ (square)}$$

$$f(x,y) = \cancel{x} + 3x^2 - x^3 + 2xy + y^2$$



$$\nabla f = \langle \cancel{1} + 6x - 3x^2 + 2y,\ 2x + 2y \rangle$$

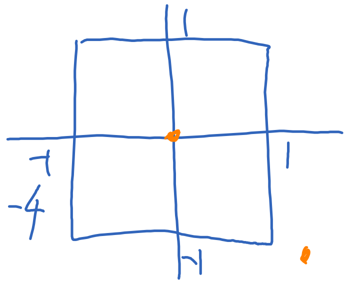
$f_{xx} = 6 - 6x$

$\nabla f = 0$ if and only if $\begin{cases} x = -y \\ 4x - 3x^2 = 0 \end{cases}$

$\frac{x}{1} + \frac{y}{-1} = 1$

$x - y = 1$

$f_{yy} = 2$



$f_{xy} = 2$

$\longrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$ or $\begin{cases} x = \frac{4}{3} \\ y = -\frac{4}{3} \end{cases}$

$D = 2(6 - 6x) - 4$

$3x^2 - x^3 + 2x(x-1) + (x-1)^2$

$= 3x^2 - x^3 + 3x^2 - 4x + 1$

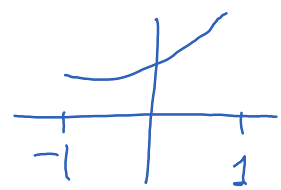
$= -x^3 + 6x^2 - 4x + 1$

@ $(0,0)$: $D = 8$

$f_{xx} = 6$

$f(0,0) = 0$

On the top edge: $x = 1$



$\boxed{-3x^2 + 12x - 4}$

$6 + 12$

$$f(1,y) = \underbrace{2 + 2y + y^2}_{g(y)}$$

$g'(y) = 2 + 2y$

min $\underline{\underline{\text{at } y=-1}}$ 3

max $\underline{\underline{\text{at } y=1}}$ 5

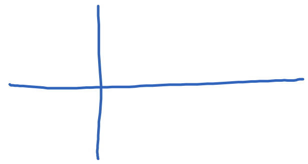
On the bottom edge: $x = -1$

$$f(-1,y) = \underbrace{4 - 2y + y^2}_{h(y)} \quad \ldots\ldots$$

Left edge: $y = -1$

$$f(x,-1) = \underbrace{3x^2 - x^3 - 2x + 1}_{k(x)}$$

$$k'(x) = -3x^2 + 6x - 2$$



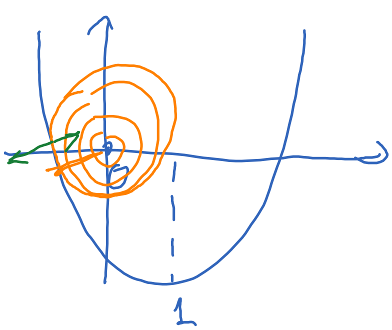
$9 \pm 3a$

$a = 0$

Optimization under constraints

$$f(x,y) \to \min/\max$$

$$g(x,y) = 0$$



$$y = (x-1)^2 - 2 = x^2 - 2x - 1$$

$$x^2 + y^2 \to \min$$

$$\underbrace{x^2 - 2x - y - 1}_{g(x,y)} = 0$$