Lecture 20

Friday, February 26, 2021 1:15 PM

* Prayer
* Spiritual thought
* Answering questions

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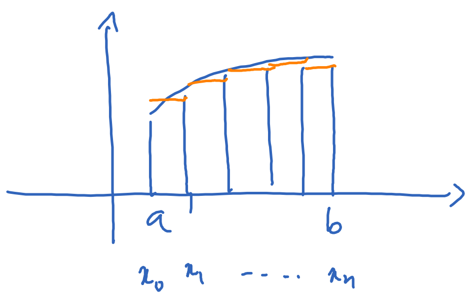
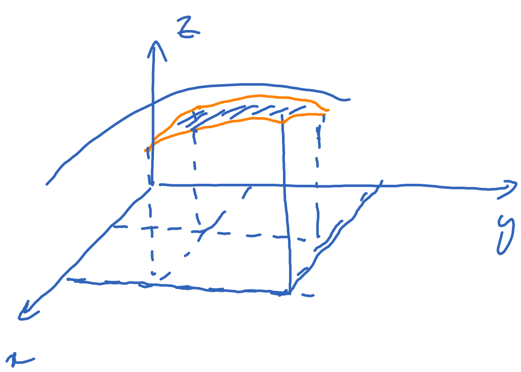
function
- limit
- derivative
  - directional derivative / rate of change
  - optimization probs.
- $\boxed{\text{integration}}$

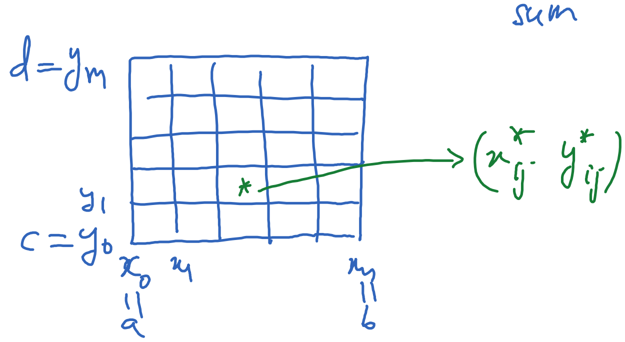


$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\underbrace{\Delta x_k}_{h} = \int_a^b f(x)\,dx.$$

Now $f = f(x,y)$

$$\iint_R f(x,y)\,dA = \lim_{m,n\to\infty} \underbrace{\sum_{j=1}^{m}\sum_{i=1}^{n} f(x_{ij}^*, y_{ij}^*)}_{\text{double Riemann sum}}$$

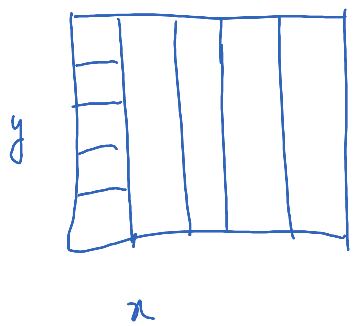




Although the definition of double integral is intuitive, it is in general not convenient to use. This leads to the notion of iterated integral.

$$\iint_{[a,b]\times[c,d]} f(x,y)\,dA \quad : \text{double integral}$$

$$\int_a^b \int_c^d f(x,y)\,dy\,dx \quad : \text{iterated integral}$$



Ex : Find the volume of the solid under the graph of

$$f(x,y) = xy$$

and above the rectangle $R = [0,2]\times[0,1]$.

Compare $\int_0^2 \int_0^1 xy\,dy\,dx$ with $\int_0^1 \int_0^2 xy\,dx\,dy$.

Fubini theorem

If f is continuous on R then the order doesn't matter

$$\int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dy\,dx \neq \int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dx\,dy$$

Average value