

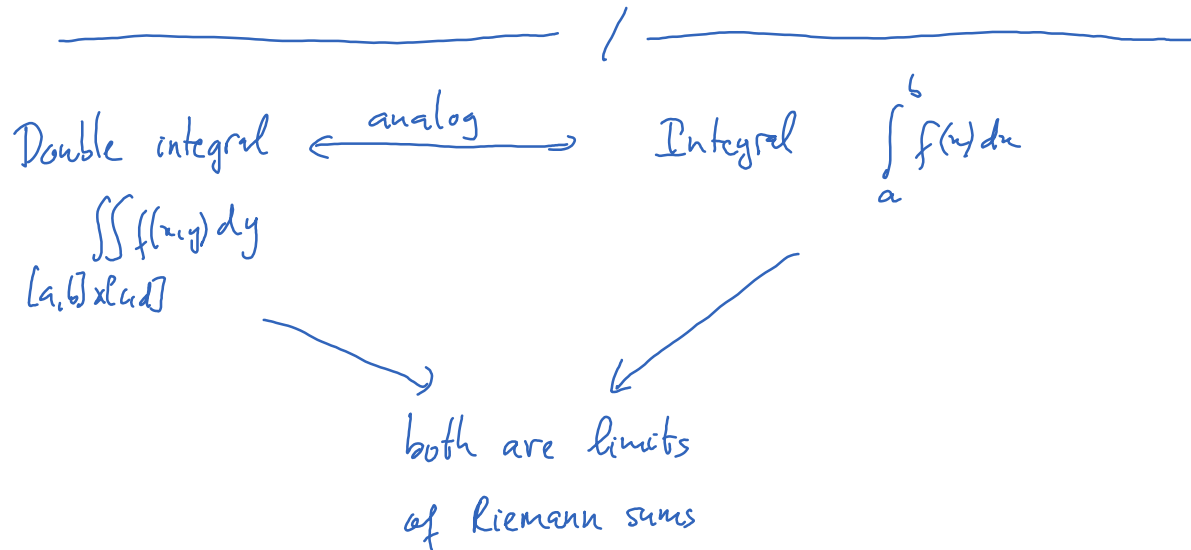
Lecture 21

Monday, March 1, 2021 2:28 PM

* Prayer

* Spiritual thought: it is not expedient that man should run faster than he has strength.

* Answering question ...



Iterated integral: a more practical way of evaluating a double integral.

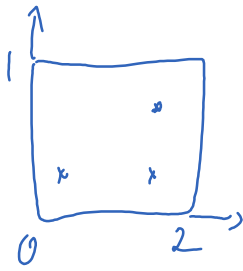
$$\underline{\text{Ex}} \quad \int_0^2 \int_0^1 xy \, dy \, dx = \int_0^2 x \frac{y^2}{2} \Big|_{y=0}^{y=1} dx = \int_0^2 \frac{x}{2} dx = \frac{x^2}{4} \Big|_0^2 = 1$$

Average value:

$$f: [a,b] \rightarrow \mathbb{R} \quad \rightsquigarrow \quad \bar{f} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$\bar{f} = \frac{1}{\text{area}(R)} \iint_R f(x,y) dA$$

Ex:



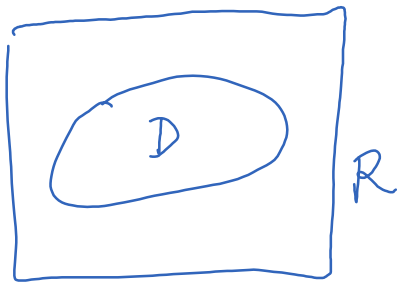
→ metal plate with mass density

$$\rho(x,y) = \frac{x+y}{3} \quad (\text{g/cm}^2)$$

$$\text{Total mass} = \iint_{[0,2] \times [0,1]} \rho(x,y) dA = \dots$$

$$\left. \frac{x^2}{2} + xy \right|_{x=0}^{x=2} \int_0^1 2+2y \quad \left. 2y+y^2 \right|_0^1$$

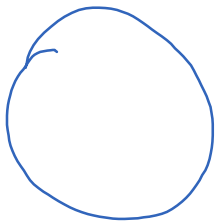
Integral over a general region



$$\iint_D f(x,y) dA \stackrel{\text{def}}{=} \iint_R \tilde{f}(x,y) dA$$

\tilde{f} is the extension of f on R by zero.

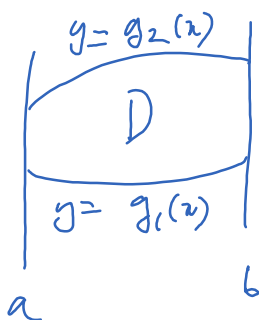
Ex



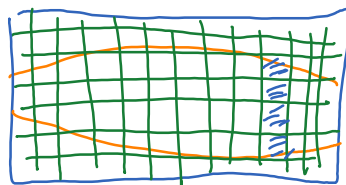
$$\rho(x,y) = x+y$$

total mass = ??

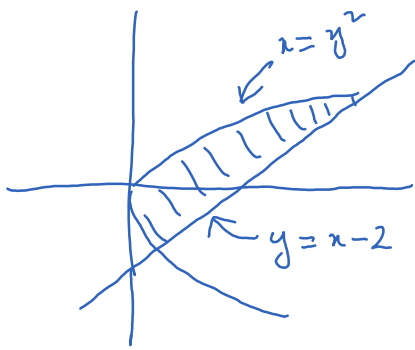
Region of Type I:



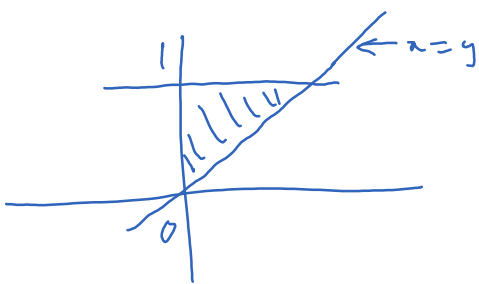
$$\iint_D f(x,y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y) dy dx.$$



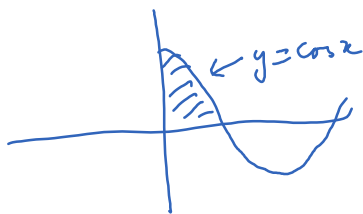
Ex :



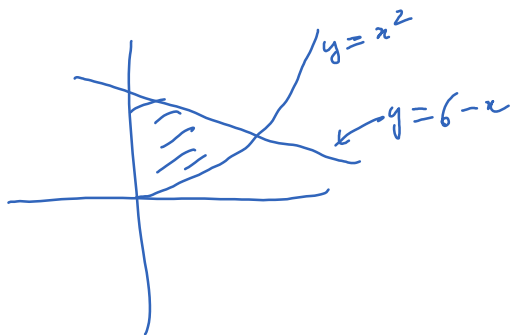
$$\iint_D y \, dA = ?$$



$$\iint_D \sin(y^2) \, dA$$



$$\iint_D x^2 \, dA$$



$$\iint_D (x+y) \, dA$$