Lecture 23

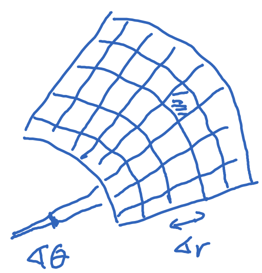
Friday, March 5, 2021 12:10 PM

* Prayer
* Spiritual thought: let God prevail → God is gathering all those who are willing to let God prevail.
* Answering questions - - - . .

Integral over a polar domain:

$a \le r \le b$

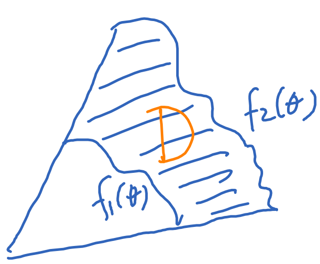
$c \le \theta \le d$



$f(r_i, \theta_j)\, r_i \Delta r \Delta\theta$

$$\iint_D f(r,\theta)\, dA = \int_c^d \int_a^b f(r,\theta)\, r\, dr\, d\theta .$$

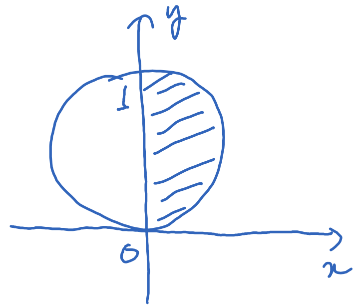
More generally,



$$D = \{(r,\theta) : c \le \theta \le d,\ f_1(\theta) \le r \le f_2(\theta)\}$$

$$\iint_D f(r,\theta)\, dA = \int_c^d \int_{f_1(\theta)}^{f_2(\theta)} f(r,\theta) \underbrace{r\, dr\, d\theta}_{dA}$$

$\underline{\underline{Ex}}$



what is the average distance to the x-axis?

$$\frac{1}{\text{area}(D)} \iint_D y \, dA$$

① $D = \left\{ (x,y) ; \ 0 \le y \le 1, \ \ 0 \le x \le \sqrt{\frac{1}{2^2} - \left(y - \frac{1}{2}\right)^2} \right\}$

$\rightsquigarrow$ difficult to integrate.

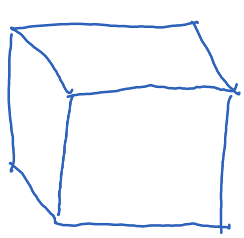
② $D = \left\{ (r,\theta) : \ 0 \le \theta \le \frac{\pi}{2}, \ \ 0 \le r \le \sin\theta \right\}$

$$\iint_D y \, dA = \int_0^{\pi/2} \int_0^{\sin\theta} r^2 \sin\theta \, dr \, d\theta = \int_0^{\pi/2} \frac{r^3}{3} \sin\theta \Big|_{r=0}^{r=\sin\theta} d\theta$$

$$= \frac{1}{3} \int_0^{\pi/2} \sin^4\theta \, d\theta \, .$$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$, ...

Triple integral

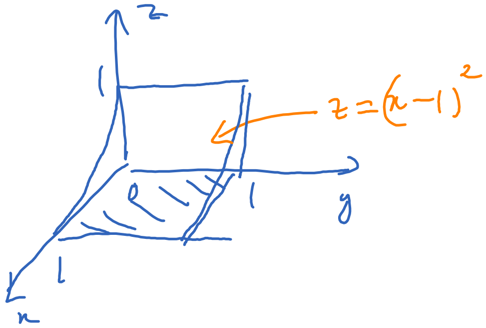


$B = \{ (x,y,z) : \ a \le x \le b, \ \ c \le y \le d, \ \ s \le z \le t \}$

$$\iiint_B f(x,y,z) \, dV = \int_a^b \int_c^d \int_s^t f(x,y,z) \, dz \, dy \, dx .$$

$\underline{\underline{Ex}}$: Box $B = [0,1] \times [0,2] \times [0,3]$ has mass density $f(x,y,z) = xyz$.

Find the total mass of the box.

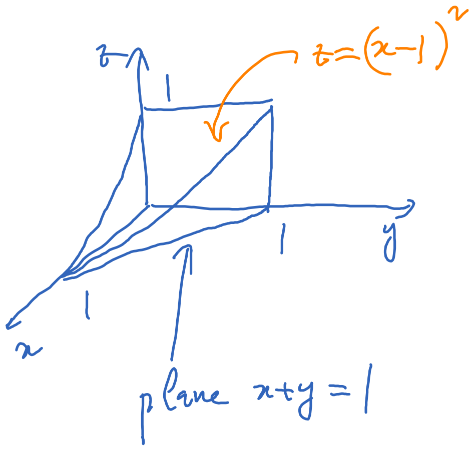


$0 \le z \le (x-1)^2$

$$\iiint_E xyz\,dV = \iint_D \int_0^{(x-1)^2} xyz\,dz\,dA$$

$$= \int_0^1\int_0^1\int_0^{(x-1)^2} xyz\,dz\,dx\,dy$$

$= \cdots\cdots$



Fix x,y. Then z runs from 0 to $(x-1)^2$.

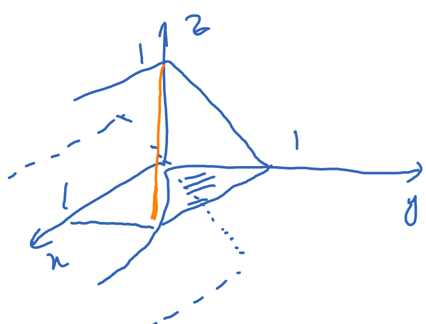
Ex

$$\int_0^1\int_{\sqrt{x}}^1\int_0^{1-y} \cdots\, dz\,dy\,dx = \int_?^?\int_?^?\int_?^? \cdots\cdots\, dx\,dy\,dz$$

$0 \le z \le 1-y \rightsquigarrow 0 \le y+z \le 1$

$\sqrt{x} \le y \le 1 \rightsquigarrow \sqrt{x} \le y \le 1$

$0 \le x \le 1 \rightsquigarrow 0 \le x \le 1$



$y=\sqrt{x}$

$y+z=1$

$z=0 \rightsquigarrow y=1 \rightsquigarrow x=1$