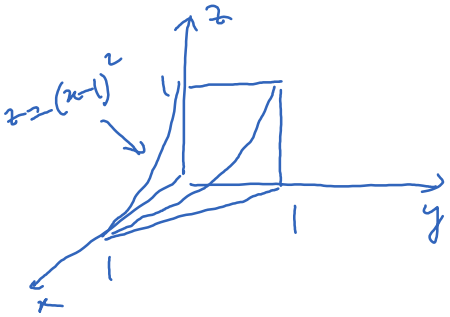
Lecture 24

Monday, March 8, 2021 2:09 PM

* Prayer
* Spiritual thought
* Answering questions....

Triple integral:



Domain is described as

$(x,y) \in D$

$0 \le z \le (x-1)^2$

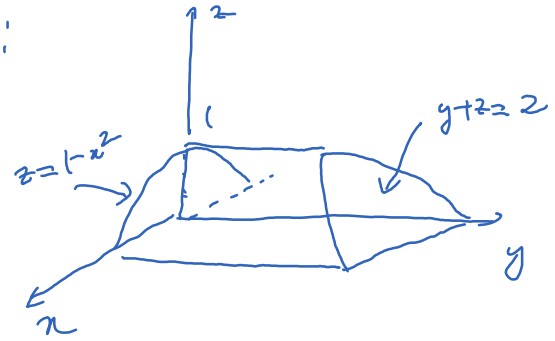
$$\text{Volume} = \iint_D \int_0^{(x-1)^2} dz\,dA = \iint_D (x-1)^2\,dA$$

$$= \int_0^1 \int_0^x (x-1)^2\,dy\,dx = \cdots$$

The mass density is $f(x,y,z) = xyz$. What is the mess?

$$\iiint_E f(x,y,z)\,dV = \int_0^1 \int_0^x \int_0^{(x-1)^2} xyz\,dz\,dy\,dx = \cdots$$

Ex:



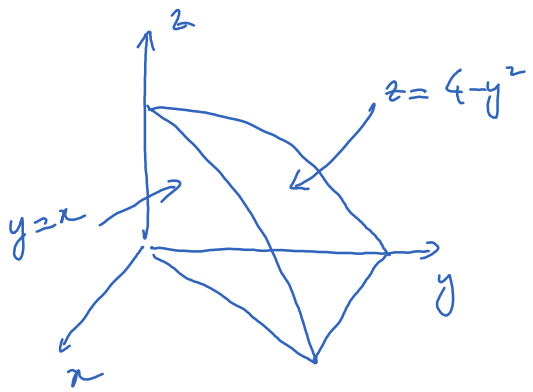
$$\iiint_E f(x,y,z)\,dV = ?$$

$$E = \{(x,z) \in D : \underbrace{g(x,z)}_{0} \le y \le \underbrace{h(x,z)}_{2-z}\}$$

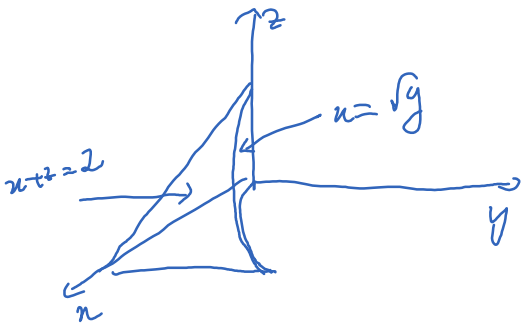
$$D = \{(x,z) : -1 \le x \le 1,\ 0 \le z \le 1-x^2\}.$$

$$\iiint_E \ldots dV = \iint_D \int_0^{2-z} \ldots \, dy\, dA = \int_{-1}^{1} \int_0^{1-x^2} \int_0^{2-z} \ldots dy\, dz\, dx.$$

Ex:



Ex:



Ex Change the order of integration

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\,dy\,dz\,dx$$

$$E = \{(x,y,z):\ 0\le x\le 1,\ 0\le y\le 1-x^2,\ 0\le z\le 1-x\}$$

bounds of y and z don't depend on each other

$$0\le y\le 1,\quad 0\le x\le \sqrt{1-y},\quad 0\le z\le 1-x$$

$$0\le z\le 1,\quad 0\le x\le 1-z,\quad 0\le y\le 1-x^2$$

Mathematica region plot ....

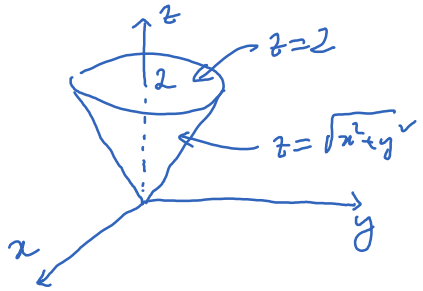
* Integral in cylindrical coordinates:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$$E = \{(x,y,z) : (x,y)\in D,\ u_1(x,y)\le z\le u_2(x,y)\}$$

$$D = \{(r,\theta):\ a\le\theta\le b,\ f_1(\theta)\le r\le f_2(\theta)\}.$$

Ex:



$$\iiint_E (x^2+y^2)\,dV$$

$$E = \{(x,y,z) : \sqrt{x^2+y^2} \le z \le 2, \ (x,y) \in D\}$$

$$D = \{(r,\theta) : 0 \le r \le 2, \ 0 \le \theta \le 2\pi\}$$

$$x^2+y^2 = r^2$$

$\sqrt{x^2+y^2} \le z \le 2$ becomes $r \le z \le 2$.

$$\iint_D \int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\, dz\, dA = \iint_D (x^2+y^2)\left(2-\sqrt{x^2+y^2}\right) dA$$

$$= \int_0^1 \int_0^{2\pi} r^2(2-r)\, r\, d\theta\, dr$$

$$= \cdots$$