Lecture 25

Wednesday, March 10, 2021 2:17 PM

* Prayer
* Spiritual thought
* Answering questions . . .

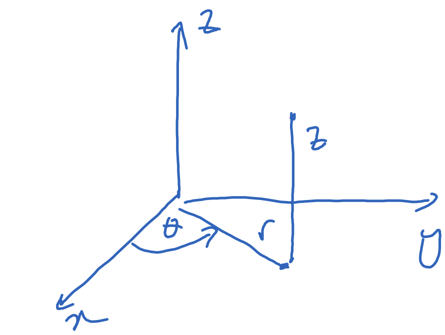
Note on triple integral:

$$\iiint_E f(x,y,z)\,dV = \iint_D \int_{g(x,y)}^{h(x,y)} f(x,y,z)\,dz\,dA$$

Determine D first, then the upper bound $h(x,y)$ and lower bound $g(x,y)$.

Don't try to find the bounds of the form $\int_?^? \int_?^? \int_?^? f(x,y,z)\,dz\,dy\,dx$.

Cylindrical coordinate:



$E: \; g(x,y) \le z \le h(x,y)$

$(x,y) \in D$

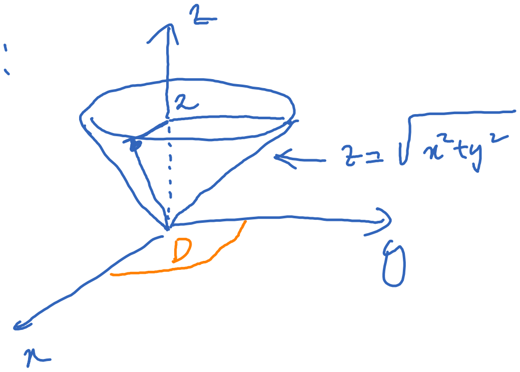
↑

polar region

$$\iiint_E f(x,y,z)\,dV = \iint_D \int_{g(x,y)}^{h(x,y)} f(x,y,z)\,dz\,dA$$

Then convert to polar coordinates from here.

Ex :



$f(x,y,z) = xz$

E is the quarter cone.

$$\iiint_E f(x,y,z)\,dV = \, ?$$

E : $(x,y) \in D$, $\sqrt{x^2+y^2} \le z \le 2$.

Change of variables

Recall :

$$\int_{[a,b]} f(x)\,dx$$

$x = g(u)$

$$\int_{[c,d]} f(g(u)) \underbrace{|g'(u)|}_{\text{"stretching factor"}} du$$

$[c,d]$ ← updated bounds

$$\iint_D f(x,y)\,dA$$

$x = g(u,v)$

$y = h(u,v)$

$$\iint_{D'} f(g(u,v), h(u,v)) \boxed{\;?\;} \, dA$$

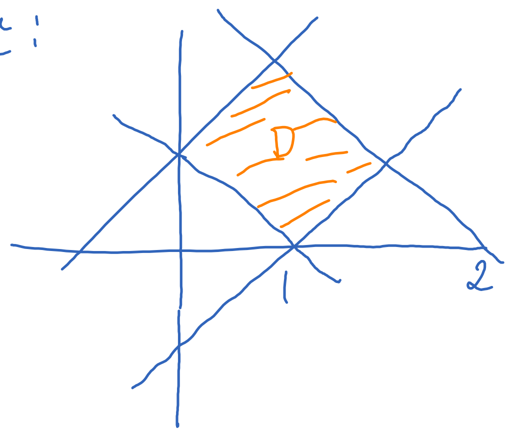
D' ← updated bounds

$\boxed{?}$ ← stretching term

Stretching term $= \left| \dfrac{\partial(x,y)}{\partial(u,v)} \right|$

$$= \left| \det \underbrace{\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}}_{\text{Jacobian matrix}} \right|$$

<u>Ex</u>:



$D = \{(x,y) : 1 \le x+y \le 2, \; -1 \le x-y \le 1 \}$

$$\iint_D x^2 \, dA = ?$$

$$\begin{cases} u = x+y \\ v = x-y \end{cases} \qquad D' = [1,2] \times [-1,1].$$

$$\rightsquigarrow \begin{cases} x = \dfrac{u+v}{2} \\ y = \dfrac{u-v}{2} \end{cases}$$

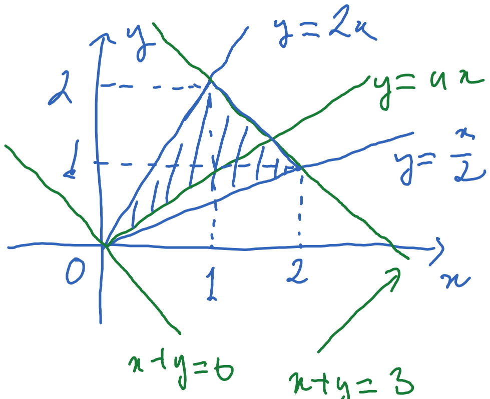
$$J = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = |\det J| = \frac{1}{2}$$

$$\iint_D \cdots = \iint_{D'} \left(\frac{u+v}{2}\right)^2 \frac{1}{2} \, dA = \int_1^2 \int_{-1}^1 \frac{(u+v)^2}{2} \frac{1}{2} \, dv \, du = \cdots$$

Ex:

$$\iint_D (x+y)\,dA = \iint_{[\frac{1}{2},2]\times[0,3]} v\,dA.$$



$$\frac{1}{2} \le u = \frac{y}{x} \le 2$$

$$0 \le v = x+y \le 3$$

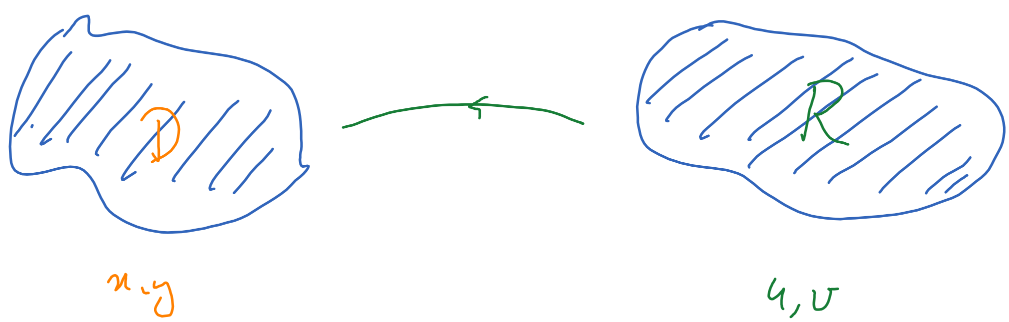
$$y = ux$$

$$v = x+y = (1+u)x$$

$$\rightsquigarrow x = \frac{v}{1+u}$$

$$y = \frac{uv}{1+u}$$

Transformation



$$x = x(u,v)$$

$$y = y(u,v)$$

$$\iint_D f(x,y)\,dA = \iint_R f(x(u,v), y(u,v))\,|\det J|\,dA.$$

Ex:

$$\int_1^2 \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx$$

$u = \frac{1}{x}$

$x = \frac{1}{u}$

$dx = -\frac{1}{u^2}\,du$

$x = \sqrt{u}$

$dx = \frac{1}{2\sqrt{u}}\,du$

$$\int_1^{1/2} u^2 \sin(u)\left(-\frac{1}{u^2}\right)du$$

$$= \int_{1/2}^{1} \sin(u)\,du = \cdots$$

Ex:

$R = [0,1]\times[0,1]$

$$\begin{cases} x = u+v \\ y = u-v \end{cases} \qquad \begin{cases} x = u-v \\ y = uv \end{cases}$$

What is the region of x, y?

Command ParametricPlot on Mathematica.