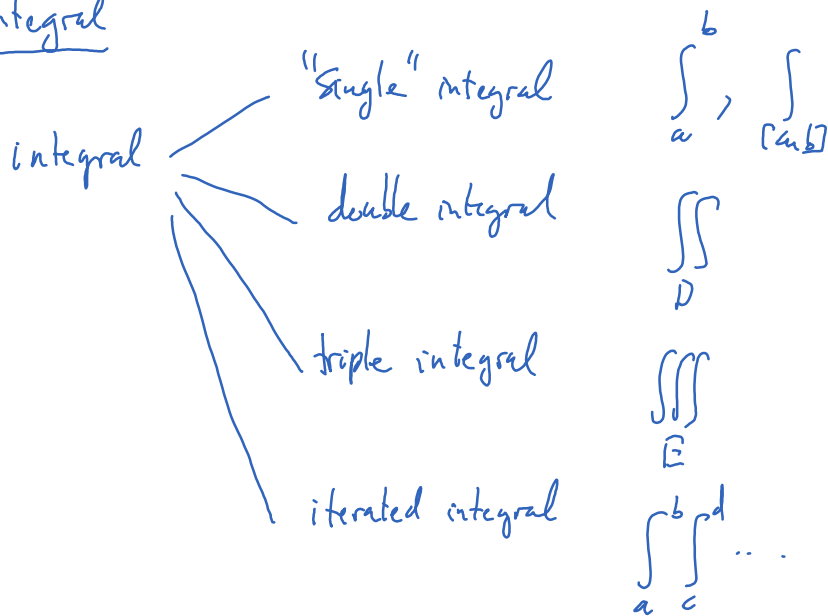


Lecture 30

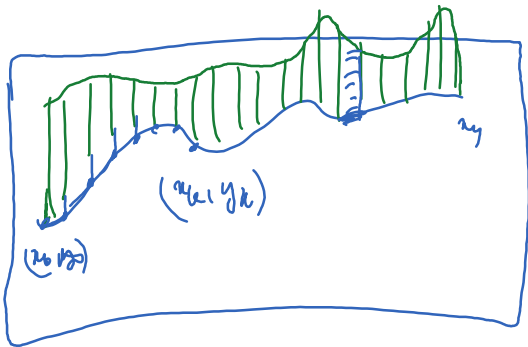
Wednesday, March 24, 2021 4:07 PM

- * Prayer
- * Spiritual thought
- * Answering questions

Line integral



Can we integrate a function $f(x,y)$ over a curve instead of a region?



$$s(t) = \int_a^t |r'(t)| dt$$

$$ds = s' dt = |r'(t)| dt$$

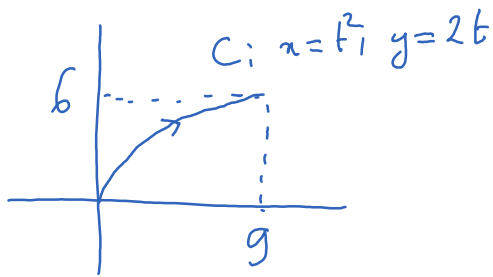
$$\int f(x_k, y_k) |r'(t_k)| dt$$

$$\approx \int_a^b f(x(t), y(t)) \underbrace{|r'(t)| dt}_{ds}$$

$$= \int_C f(x,y) ds$$

integral of a scalar function over a curve.

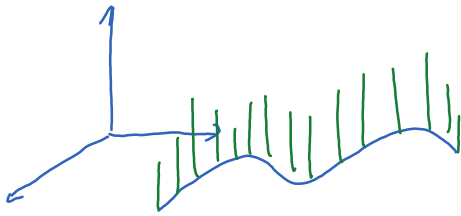
Ex



$$\int_C y ds = ?$$

Mathematics:

how to plot function $f(x,y)$ on top of a curve C ?



$$C: (x(t), y(t), 0)$$

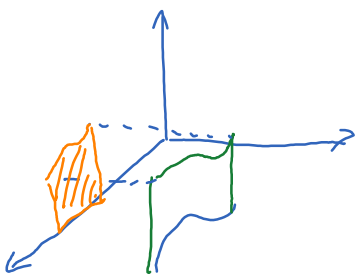
$$\text{Surface: } (x(t), y(t), s)$$

$$t \in [a, b], s \in [0, f(x(t), y(t))]$$

Parametric Plot 3D $[\{t^2, 2t, s\}, \{t, 0, 3\}, \{s, 0, 2t\}]$

$\int_C f(x,y) ds$ is the area of the wall.
or mass of a wire

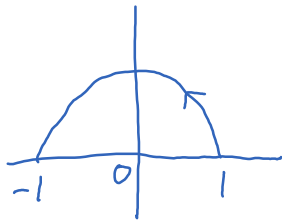
* Line integral writ x or y



$$\int_C f(x,y) dx = \text{area of the shadow of the fence.}$$

Ex

$$f(x,y) = x+y$$



$$\int_C (x+y) dx = \int_0^{2\pi} (\cos t + \sin t)(-\sin t) dt$$

$$= \dots$$

Line integral of a vector field

Work along a curve



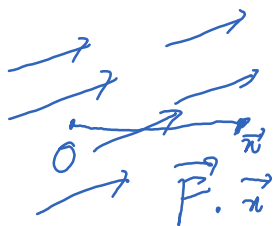
$$\int \vec{F} \cdot d\vec{r} = \int \vec{F} \cdot \frac{d\vec{r}}{|\vec{r}'|} |\vec{r}'|$$

$$\frac{\vec{F} \cdot d\vec{r}}{|\vec{r}'|}$$



$$\int \frac{\vec{F} \cdot \vec{r}'}{|\vec{r}'|} ds$$

$$\frac{\vec{r}'}{|\vec{r}'|} ds = \frac{r' dt}{|r'| dt} = dr$$



Ex

$$F(x,y,z) = \langle xz, y, 1 \rangle$$

Along the curve C: $\begin{cases} x = t \\ y = 2t \\ z = t^2 \end{cases} \quad 0 \leq t \leq 1$