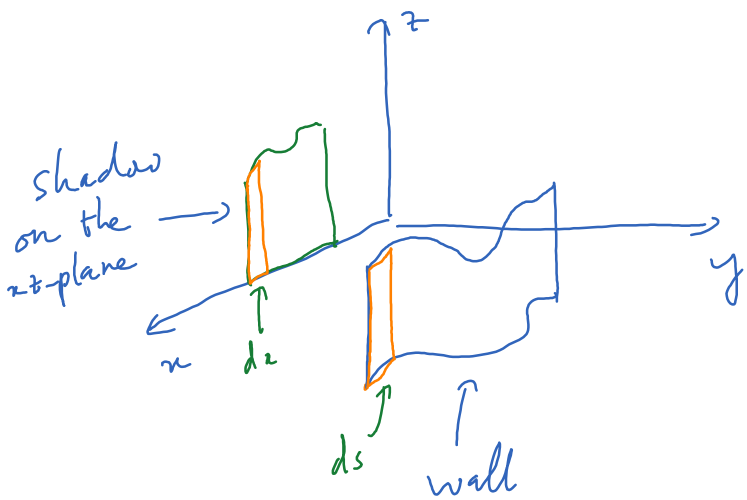
Lecture 31

Friday, March 26, 2021 1:55 PM

* Prayer
* Spiritual thought
* Answering questions ....

Line integrals

$$\underbrace{\int_C f(x,y)\,ds}_{\text{a self-contained notation}} = \int_a^b f(x(t),y(t))\underbrace{\sqrt{x'^2+y'^2}\,dt}_{ds}$$

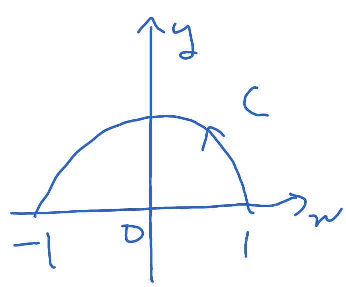


Area of the shadow

$$= \int_C f(x,y)\,dx$$

$$= \int_a^b f(x(t),y(t))\,x'(t)\,dt$$

Ex

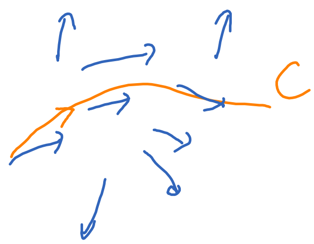


$$\int_C (x+y)\,dy = ?$$

$$C: \begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad 0 \le t \le \pi$$

$$\int_C (x+y)\,\underbrace{dy}_{y'dt} = \int_0^{\pi} (\cos t + \sin t)\cos t\,dt = \cdots$$

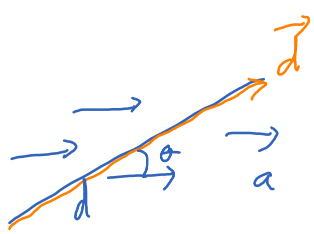
Line integral of a vector field



Imagine a particle moving along a curve C inside a force field F.

How to quantify the influence of the force field on the particle?

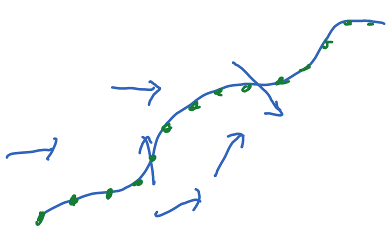
To simplify the problem, assume F is a constant vector field on C is a straight line.



$F(x,y) = \underset{\text{vector}}{a} \quad \forall x,y$

Work done by F is $|\vec{a}|\, d \cos\theta$

$= \vec{a} \cdot \vec{d}$



On each segment, F can be viewed as constant and C can be view as a straight line segment. Work done $= F(r) \cdot dr$

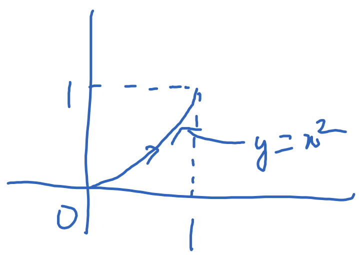
Total work along C = $\underbrace{\int_C F \cdot dr}$

line integral of vector field F along C.

How to compute?

$$\int_C F \cdot dr = \int_a^b F(r) \cdot r' \, dt$$

Ex



$F(x,y) = \langle -y, x \rangle$

$C: \begin{cases} x = t \\ y = t^2 \end{cases} \quad (0 \le t \le 1)$

$$\int_C F \cdot dr = \int_0^1 F(r(t)) \cdot r'(t) \, dt$$

$r(t) = \langle t, t^2 \rangle$

$r'(t) = \langle 1, 2t \rangle$

$F(r(t)) = F(t, t^2) = \langle -t^2, t \rangle$

$$\int_C F \cdot dr = \int_0^1 \langle -t^2, t \rangle \cdot \langle 1, 2t \rangle \, dt = \int_0^1 (-t^2 + 2t^2) \, dt = \frac{1}{3}.$$

Note: If we write $\underset{x,y}{F} = \langle \underset{x,y}{P}, \underset{x,y}{Q} \rangle$ then

$$\int_C F \cdot dr = \int_C \underbrace{\langle P, Q \rangle}_{F} \cdot \underbrace{\langle dx, dy \rangle}_{dr}$$

$$= \underbrace{\int_C P(x,y)\,dx + Q(x,y)\,dy}_{\text{quite a common notation in the textbook and other places.}}$$