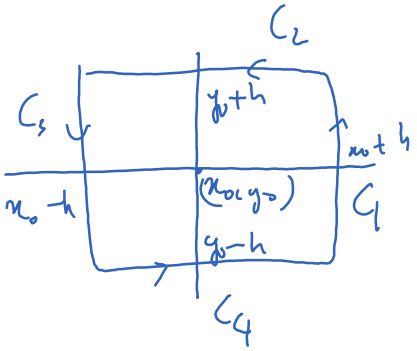
Lecture 33

Tuesday, March 30, 2021 4:08 PM

* Prayer
* Spiritual thought
* Answering questions....

Green's theorem



$$C_1: \begin{cases} x = x_0 + h \\ y = y_0 + t \end{cases} \quad -h \le t \le h$$

$$C_2: \begin{cases} x = x_0 - t \\ y = y_0 + h \end{cases} \quad -h \le t \le h$$

$$C_3: \begin{cases} x = x_0 - h \\ y = y_0 - t \end{cases} \quad -h \le t \le h$$

$$C_4: \begin{cases} x = x_0 + t \\ y = y_0 - h \end{cases} \quad -h \le t \le h$$

$$\text{Circulation} = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$$

$$= \int_{C_1} \underbrace{L}_{0}\,dx + Q\,dy + \int_{C_2} \underbrace{L}_{0}\,dx + \underbrace{Q}_{0}\,dy + \int_{C_3} \underbrace{P}_{0}\,dx + Q\,dy + \int_{C_4} P\,dx + \underbrace{Q}_{0}\,dy$$

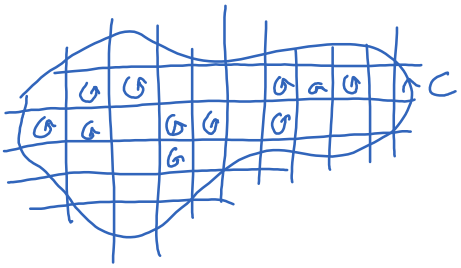
$$= \int_{C_1} Q\,dy + \int_{C_2} P\,dx + \int_{C_3} Q\,dy + \int_{C_4} \underline{P}\,dx$$

$$= \int_{-h}^{h} \left[Q(x_0+h, y_0+t) - Q(x_0-h, y_0+t) \right] dt - \int_{-h}^{h} \left[P(x_0+t, y_0+h) - P(x_0-t, y_0+h) \right] dt$$

$$\approx 4h^2 Q_x(x_0, y_0) - 4h^2 P_y(x_0, y_0)$$

Circulation density = $\frac{\text{total circulation}}{\text{area} = \Delta A}$

$= Q_x(x_0, y_0) - P_y(x_0, y_0)$.



Total circulation

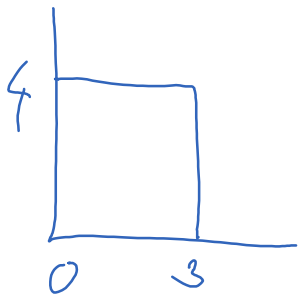
$= \int_C F \cdot dr$

Total circulation $= \iint_D (Q_x - P_y)\, dA$

Green's theorem:

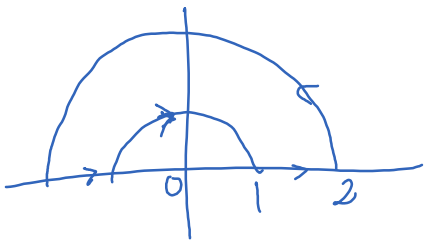
$$\int_C F \cdot dr = \iint_D (Q_x - P_x)\, dA$$

Ex



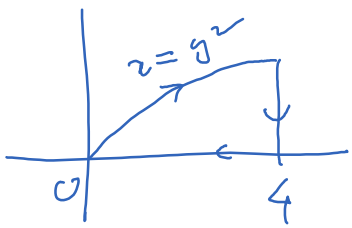
$\int_C ye^x dx + 2e^x dy$

Ex



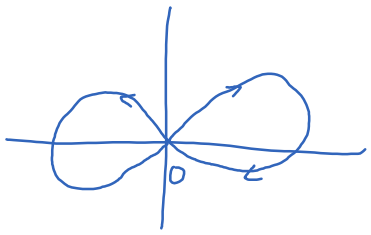
$\int_C y^2 dx + 3xy\, dy$

Ex



$\int_C (x^{2/3} + y^2)\, dx + (y^{4/3} - x^2)\, dy$

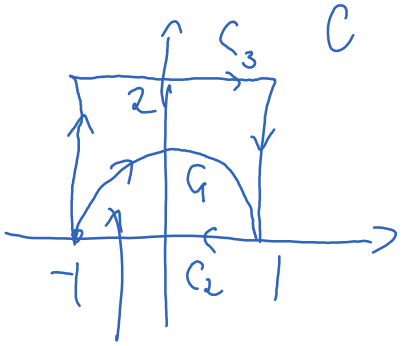
Ex



$C: \begin{cases} x = \sin t \\ y = \sin t \cos t \end{cases}$ $\quad 0 \le t \le 2\pi$

$$\int_C x\,dx + y\,dy$$

Ex



$y = 1 - x^2$

$C = C_1 + C_2 + C_3 + C_2$

$= (C_1 + C_2) + (C_3 + C_2)$

$$\int_C x\,dx + y\,dy = ?$$