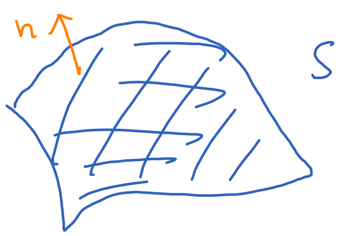
Lecture 37

Friday, April 9, 2021 2:37 PM

* Prayer
* Spiritual thought
* Answering questions....

Surface normal vectors

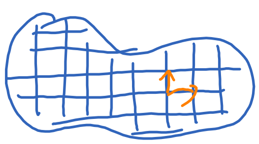


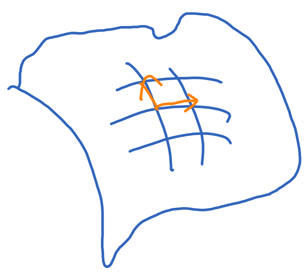
Curve : $r(t)$

tangent vector : $r'(t) = \langle x(t), y(t) \rangle$

normal vector : $n(t) = \langle y(t), -x(t) \rangle$

binormal vector : ....





$$n = \frac{r_u \times r_v}{|r_u \times r_v|}$$

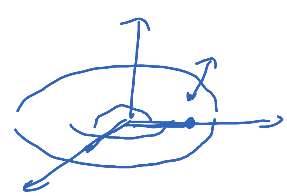
Ex Torus

$$\begin{cases} x = (2+\cos\phi)\cos\theta \\ y = (2+\cos\phi)\sin\theta \\ z = \sin\phi \end{cases}$$

$0 \le \theta \le 2\pi$

$0 \le \phi \le 2\pi$

Normal vector at $(0, 2, 1)$ is ?



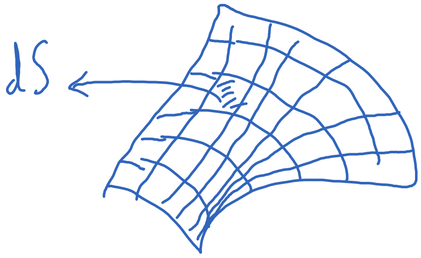
$\phi = \theta = \frac{\pi}{2}$

$r_\phi = \langle -\sin\phi\cos\theta, -\sin\phi\sin\theta, \cos\phi \rangle = \langle \dots \rangle$

$r_\theta = \langle -(2+\cos\phi)\sin\theta, (2+\cos\phi)\cos\theta, 0 \rangle = \langle \dots \rangle$

$\boxed{r_\phi \times r_\theta}$

Surface integral



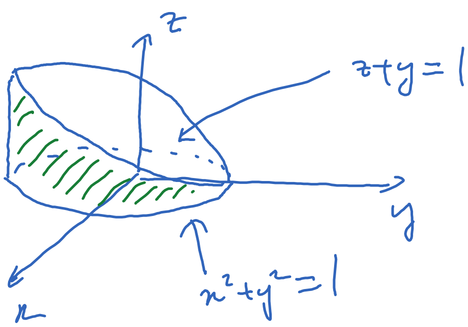
line integral
- of a scalar func : area of fence, weight of a wire
- of a vector field : work, Circulation

surface integral
- of a scalar function : weight of a surface
- of a vector field : flux

f : mass density

$$\text{total mass} \approx \sum f \, \Delta S = \iint_S f \, dS = \iint_D f(\mathbf{r}) |\mathbf{r}_u \times \mathbf{r}_v| \, dA.$$

Ex:



$f(x,y,z) = xy$ ← mass density (per unit area)

$$\iint_S f \, dS = ?$$

$$S : \begin{cases} x = \cos\theta \\ y = \sin\theta \\ z = z \end{cases} \qquad \underbrace{\begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le z \le 1 - \sin\theta \end{matrix}}_{D}$$

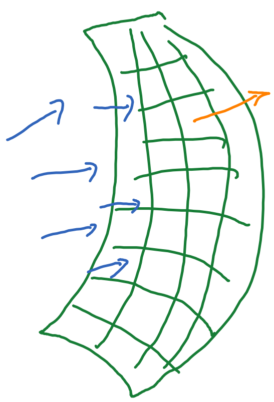
$$\left.\begin{matrix} \mathbf{r}_\theta = \langle x_\theta, y_\theta, z_\theta \rangle = \langle -\sin\theta, \cos\theta, 0 \rangle \\ \mathbf{r}_z = \langle x_z, y_z, z_z \rangle = \langle 0, 0, 1 \rangle \end{matrix}\right\} \quad \mathbf{r}_\theta \times \mathbf{r}_z = \langle \cos\theta, \sin\theta, 0 \rangle$$

$$\iint_D f(r)\,\underbrace{|r_\theta \times r_z|}_{1}\,dA = \iint_S xy\,dA$$

$$= \int_0^{2\pi}\int_0^{1-\sin\theta} \cos\theta\sin\theta\,dz\,d\theta$$

$$= \cdots$$

Surface integral of a vector field



Flux across S is $\iint \vec{F}\cdot\vec{n}\,dS$

$d\vec{S} = \vec{n}\,dS$

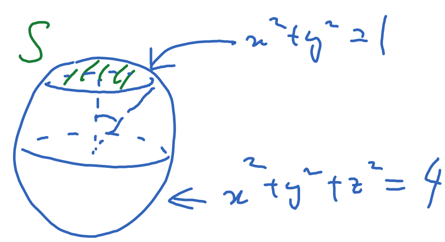
$$\iint_S \vec{F}\cdot d\vec{S}$$

How to compute:

$$\vec{n} = \frac{r_u \times r_v}{|r_u \times r_v|}$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot\vec{n}\,dS = \iint_D \vec{F}\cdot(r_u \times r_v)\,dA.$$

Ex: $\vec{F} = \langle xy, z, x\rangle$



$$S: \begin{cases} x = \sin\phi\cos\theta \\ y = \sin\phi\sin\theta \\ z = \cos\phi \end{cases} \qquad \underbrace{\begin{array}{c} 0 \le \phi \le \frac{\pi}{6} \\ 0 \le \theta \le 2\pi \end{array}}_{D}$$

$$r_\phi = \langle x_\phi, y_\phi, z_\phi \rangle$$

$$= \langle \cos\phi\cos\theta, \cos\phi\sin\theta, -\sin\phi \rangle$$

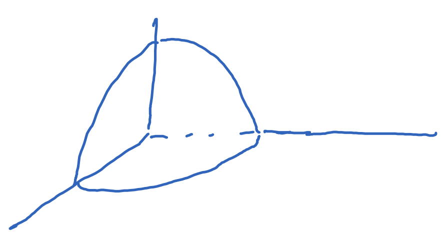
$$r_\theta = \langle -\sin\phi\sin\theta, \sin\phi\cos\theta, 0 \rangle$$

$$r_\phi \times r_\theta = \langle \sin^2\phi\cos\theta, \sin^2\phi\sin\theta, \cos\phi\sin\phi \rangle$$

$$r_\phi \times r_\theta = \langle \sin\phi\cos\phi \ldots \rangle$$

$$\iint_S \vec{F} \cdot dS = \iint_D \ldots$$

Ex



The Mobius strip:

$$\begin{cases} x = 2\cos\theta + r\cos\frac{\theta}{2} \\ y = 2\sin\theta + r\cos\frac{\theta}{2} \\ z = r\sin\frac{\theta}{2} \end{cases} \qquad \begin{array}{l} 0 \le \theta \le 2\pi \\ -\frac{1}{2} \le r \le \frac{1}{2} \end{array}$$

Find normal vector at ....