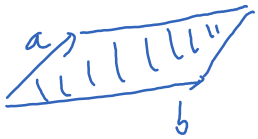
Lecture 4

Wednesday, January 20, 2021 1:22 PM

* Prayer
* Spiritual thought : "I will bless all those who labor in my vineyard with a mighty blessing..." (D&C 21:9)
* Answering questions...

Cross product

$a \times b$



$a \times b \perp a, b$

$|a \times b|$ = area of parallelogram

* Question: $a \times b \stackrel{?}{=} b \times a$ (no)

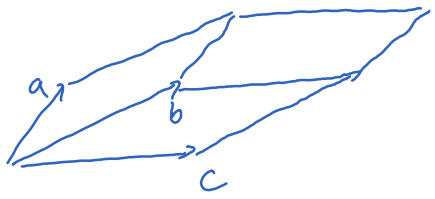
$a \times (b \times c) \stackrel{?}{=} (a \times b) \times c$ (no)

* Cross product can be used to check if 3 points are colinear:

$\vec{AB} \times \vec{AC} = 0$ if and only if A, B, C are colinear.

* Triple product: $a \cdot (b \times c)$

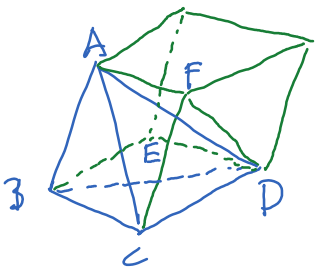
Another representation: $a \cdot (b \times c) = \det \begin{bmatrix} — a — \\ — b — \\ — c — \end{bmatrix} = \det \begin{bmatrix} | & | & | \\ a & b & c \\ | & | & | \end{bmatrix}$



Geometrically,

$|a \cdot (b \times c)|$ = volume of the parallelopiped formed by a, b, c.

Ex



$A(1,0,0)$

$B(2,1,0)$

$C(3,1,1)$

$D(0,2,-1)$

$vol(ABCD) = vol(ABDE) = vol(ACDF)$

Sum $= \frac{1}{2}$ volume of parallelopiped

Thus,

$vol(ABCD) = \frac{1}{6}$ vol (parallelopiped)

$= \frac{1}{6} \left| \overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) \right|$

$= \frac{1}{6} \left| \langle 1,1,0 \rangle \cdot (\langle 2,1,1 \rangle \times \langle -1,2,-1 \rangle) \right|$

$= \frac{1}{6} \left| \langle 1,1,0 \rangle \cdot \langle -3,1,5 \rangle \right|$

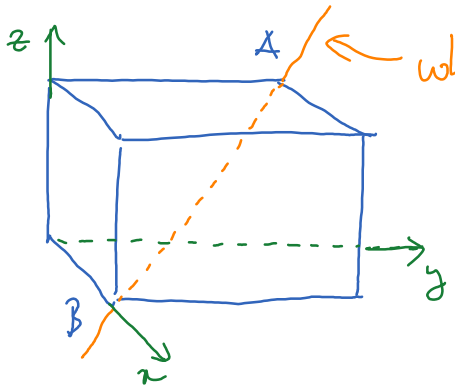
$= \frac{1}{6} |-3+1+0| = \frac{1}{3}$

2 1 1 2 1
−1 2 −1 −1 2

Equation of lines

- Vector equation $\longrightarrow \vec{r} = \vec{r}_0 + t\vec{a}$
- parametric equation $\rightsquigarrow \begin{cases} x = x_0 + ta_1 \\ y = y_0 + ta_2 \\ z = z_0 + ta_3 \end{cases}$
- symmetric equation $\rightsquigarrow \dfrac{x-x_0}{a_1} = \dfrac{y-y_0}{a_2} = \dfrac{z-z_0}{a_3}$

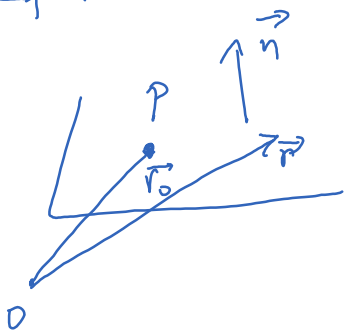
Ex



what is the equation of this line?

what is the equation of the line segment from A to B?

Equation of planes



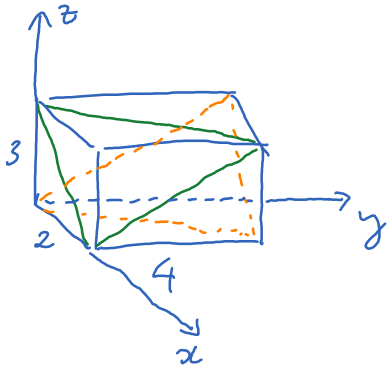
$$(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$$

vector eq.

Scalar equation $\quad ax + by + cz = ax_0 + by_0 + cz_0$

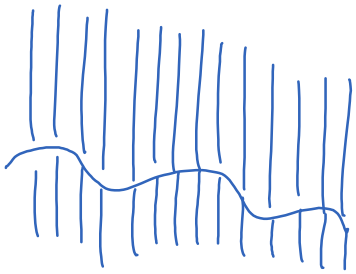
where $\vec{n} = \langle a, b, c \rangle$ and $P(x_0, y_0, z_0)$.

Ex



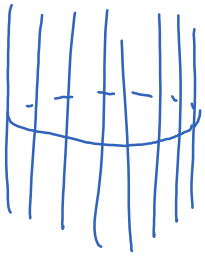
- Find the equation of the green plane.
- Find the equation of the red plane
- Find the equation of the intersections of the two planes.

Cylinder surfaces



A cylinder surface is a "wall" built on a curve.

circular cylinder



elliptic cylinder

parabolic cylinder.

Quadratic surfaces are of the form $Ax^2+By^2+Cz^2+Dyz+Exz+Fxy+Gx+Hy+Iz+J=0$

Let $u=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

$u^T Q u = Pu$

symmetric

diagonalizable

$$v^T \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} v = Rv$$

$$\lambda_1 x^2+\lambda_2 y^2+\lambda_3 z^2 = r_1 x+r_2 y+r_3 z$$

$$\lambda_1\left(x-\frac{r_1}{2\lambda_1}\right)^2+\lambda_2\left(y-\frac{r_2}{2\lambda_2}\right)^2+\lambda_3 z^2 = \#+r_3 z$$

In most cases, the equation can be "reduced" to

$$Ax^2+By^2+Cz^2+J=0$$

$$Ax^2+By^2+Cz+J=0$$

Plotting with Mathematica:

Plot3D, ContourPlot3D