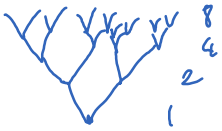
Lecture 5

Friday, January 22, 2021 2:04 PM

* Prayer
* Spiritual thought:

Science, if studied in faith, is very helpful.

Growth of bacteria:



Population at generation n is 2^n

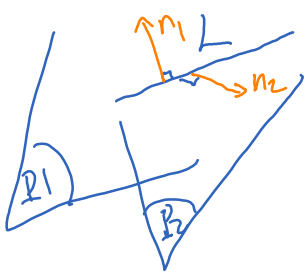
Total population up to generation n is $2^n - 1$.

* Answering questions ...

Intersection of two planes:

Plane 1: $6x - 3y + 2z = 1$

Plane 2: $2x + y + 4z = 2$



Normal vector of plane 1 is $n_1 = \langle 6, -3, 2 \rangle$.

" " 2 " $n_2 = \langle 2, 1, 4 \rangle$.

L has direction vector $n_1 \times n_2 = \langle -14, -20, 12 \rangle$.

We can choose a direction vector $a = \langle -7, -10, 6 \rangle$.

We then need to find one point on L:

$$\begin{cases} 6x-3y+2z=1 \\ 2x+y+4z=2 \end{cases}$$

Choose $z=0$: $\begin{cases} 6x-3y=1 \\ 2x+y=2 \end{cases} \rightsquigarrow \begin{cases} x=7/12 \\ y=5/6 \end{cases}$

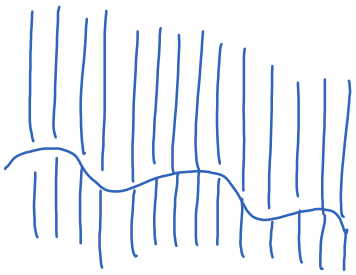
L passes through $A\left(\frac{7}{12}, \frac{5}{6}, 0\right)$.

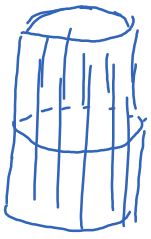
Equation of L is

$$\begin{cases} x=\frac{7}{12}-7t \\ y=\frac{5}{6}-10t \\ z=0+6t \end{cases}$$

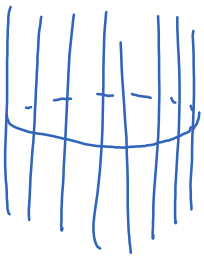
Cylinder surfaces

A cylinder surface is a "wall" built on a curve.





circular cylinder



elliptic cylinder

parabolic cylinder.

Quadratic surfaces are of the form $Ax^2+By^2+Cz^2+Dyz+Exz+Fxy+Gx+Hy+Iz+J=0$

Let $u=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

$$u^T Q u = Pu$$

symmetric

diagonalizable

$$v^T \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} v = Rv$$

$$\lambda_1 x^2+\lambda_2 y^2+\lambda_3 z^2 = r_1 x+r_2 y+r_3 z$$

$$\lambda_1\left(x-\frac{r_1}{2\lambda_1}\right)^2+\lambda_2\left(y-\frac{r_2}{2\lambda_2}\right)^2+\lambda_3 z^2 = \#+r_3 z$$

In most cases, the equation can be "reduced" to

$$Ax^2+By^2+Cz^2+J=0$$

$$Ax^2+By^2+Cz+J=0$$

Plotting with Mathematica:

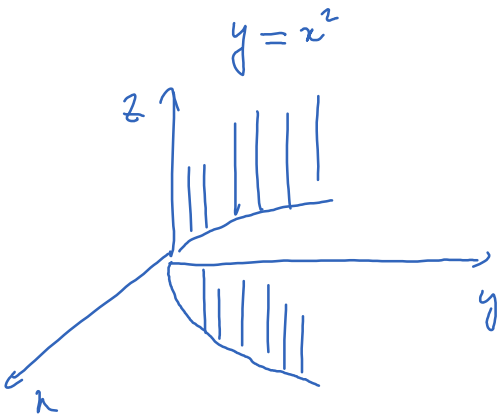
Plot3D, ContourPlot3D

Describe traces on planes....

Vector functions

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ (sometimes the arrow on $\vec{r}$ is dropped.)

Ex $\vec{r}(t) = \langle t, t^2, t^3 \rangle$



The curve is contained in the cylindrical surface $y = x^2$.

On this surface, $z = x^3$.

That's why the curve is called a "twisted cubic".

Ex:

$\vec{r}(t) = \langle t\cos t, t\sin t, t \rangle$