Lecture 8

Friday, January 29, 2021 2:11 PM

* Prayer

* Spiritual thought:

* Answering questions ....

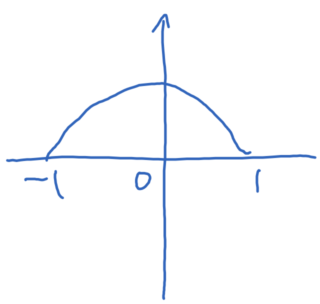
Length of a curve:

$$L = \int_a^b |r'(t)|\,dt$$

Note: length is independent of the way a curve is parametrized.

In other words, it is an "intrinsic" property of a curve.

Ex

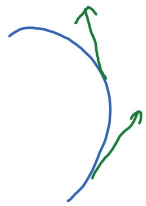


half the circle

$$r_1: \begin{cases} x = \cos t \\ y = \sin t \\ z = 0 \end{cases} \qquad r_2: \begin{cases} x = t \\ y = \sqrt{1-t^2} \\ z = 0 \end{cases}$$

$$L_1 = \int_0^{\pi} |r_1'(t)|\,dt = \pi, \qquad L_2 = \int_{-1}^{1} \frac{1}{\sqrt{1-t^2}}\,dt = \pi$$

Curvature: this is another intrinsic property of a curve.



It measures how sharply the curve bends at a given point.

$\cancel{\kappa'(t) = |r''(t)|}$ ← this definition would make the curvature depend on the parametrization.

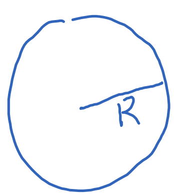
Instead,

$$\kappa(t) = \left|\frac{dT}{ds}\right| = \frac{|T'(t)|}{|r'(t)|}.$$

An equivalent formula is

$$\kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}.$$

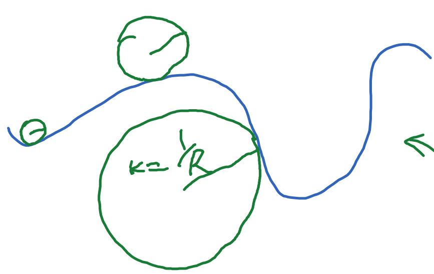
How to interpret the curvature?



$\kappa(t) = \frac{1}{R}$ for the circle with radius R.

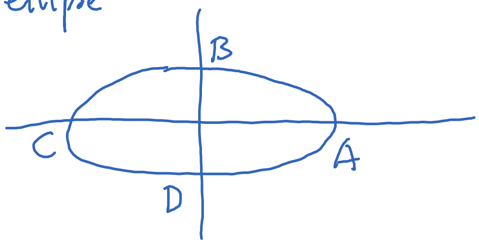
R large → κ is small

R small → κ is large



← circle of curvature is the circle that best fits the curve at a given point.

Ex the ellipse

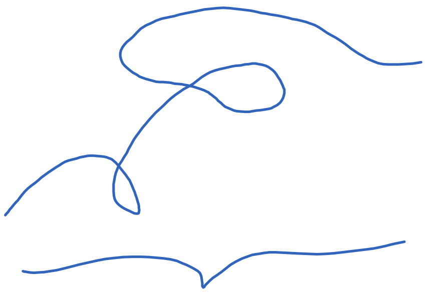


$$x^2 + 4y^2 = 4$$

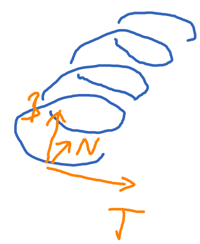
What are the curvatures at A, B, C, D?

$$r(t): \begin{cases} x = 2\cos t \\ y = \sin t \\ z = 0 \end{cases}$$

<u>Torsion</u>



plane curve ; no torsion

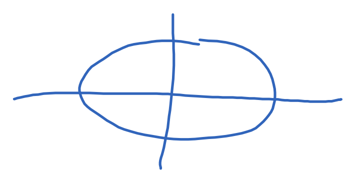


$$N(t) = \frac{T'(t)}{|T'(t)|} \quad \text{principal normal vector}$$

$$B(t) = T(t) \times N(t) \quad \text{binormal vector}$$

$$T(t) = \frac{v(t)}{|v(t)|}$$

$$a(t) = \underbrace{|v(t)|'}_{\text{tangential acce.}} T + \underbrace{k|v(t)|^2}_{\text{normal acce.}} N$$



drive around the ellipse.