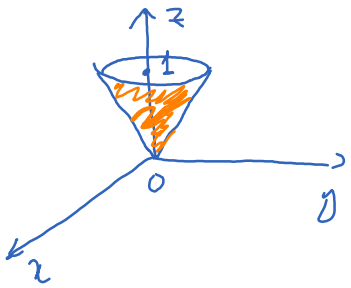
Ex Find surface area of the cone $z^2 = x^2 + y^2$ between $z = 0$ and $z = 1$.



Two ways to parametrize this cone:

(I): $\begin{cases} x = u \\ y = v \\ z = \sqrt{u^2+v^2} \end{cases}$, $(u,v) \in$ unit disk

(II): $\begin{cases} x = v\cos u \\ y = v \sin u \\ z = v \end{cases}$ $0 \le v \le 1$, $0 \le u \le 2\pi$

Using parametrization (I)

$$r_u = (x_u, y_u, z_u) = \left(1, 0, \frac{u}{\sqrt{u^2+v^2}}\right)$$

$$r_v = (x_v, y_v, z_v) = \left(0, 1, \frac{v}{\sqrt{u^2+v^2}}\right)$$

$$r_u \times r_v = \left(\frac{-u}{\sqrt{u^2+v^2}}, \frac{-v}{\sqrt{u^2+v^2}}, 1\right)$$

$$|r_u \times r_v| = \sqrt{\frac{u^2}{u^2+v^2} + \frac{v^2}{u^2+v^2} + 1} = \sqrt{2}$$

Area of surface $= \iint_{\text{disk}} |r_u \times r_v| \, dA = \iint_{\text{disk}} \sqrt{2} \, dA = \sqrt{2} \cdot \text{area(disk)} = \sqrt{2}\pi$

Using parametrization (II)

$$r = (v\cos u, v\sin u, v)$$

$$\left.\begin{aligned} r_u &= (-v\sin u, v\cos u, 0) \\ r_v &= (\cos u, \sin u, 1) \end{aligned}\right\} \quad r_u \times r_v = (v\cos u, v\sin u, -v)$$

$$|r_u \times r_v| = \sqrt{v^2\cos^2 u + v^2\sin^2 u + v^2} = \sqrt{2v^2} = v\sqrt{2}.$$

area of S is

$$\int_0^1 \int_0^{2\pi} |r_u \times r_v| \, du\, dv = \int_0^1 \int_0^{2\pi} v\sqrt{2} \, du\, dv = \pi\sqrt{2}$$