Problem 14 of 14.8:

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$$ where $f(x,y,z) = x^4 + y^4 + z^4$,

$g(x,y,z) = x^2 + y^2 + z^2 - 1$

The system becomes:

$$\begin{cases} 4x^3 = 2\lambda x \\ 4y^3 = 2\lambda y \\ 4z^3 = 2\lambda z \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

This is a system of 4 equations and 4 unknowns.

Equivalently,

$$\begin{cases} 2x(2x^2 - \lambda) = 0 & \rightsquigarrow & x = 0 \ \underline{\underline{\text{or}}} \ \lambda = 2x^2 \\ 2y(2y^2 - \lambda) = 0 & \rightsquigarrow & y = 0 \ \underline{\underline{\text{or}}} \ \lambda = 2y^2 \\ 2z(2z^2 - \lambda) = 0 & \rightsquigarrow & z = 0 \ \underline{\underline{\text{or}}} \ \lambda = 2z^2 \\ x^2 + y^2 + z^2 = 1 & \rightsquigarrow & x^2 + y^2 + z^2 = 1 \end{cases}$$

There are 8 cases total:

(1) $x = 0, \ y = 0, \ z = 0 \longrightarrow$ excluded because $x^2 + y^2 + z^2 = 0 \neq 1$

(2) $x = 0, \ y = 0, \ \lambda = 2z^2 \rightsquigarrow z^2 = 1 - x^2 - y^2 = 1 \rightsquigarrow x = 0, y = 0, z = \pm 1$

(3) $x = 0, \ \lambda = 2y^2, \ \lambda = 2z^2$

- - - - -

(8) $\lambda = 2x^2, \ \lambda = 2y^2, \ \lambda = 2z^2 \rightsquigarrow x^2 = y^2 = z^2 = 1/3 \rightsquigarrow \ldots\ldots$

In total, you will see that there are 26 triples of (x,y,z). The maximum of f at these points is the maximum.