Lecture 10

Tuesday, January 25, 2022 9:12 PM

* Prayer
* Spiritual thought

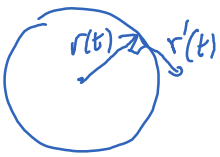
Think about gliding on a roller coaster.

Moving frame: T, N, B

$$N = \frac{T'(t)}{|T'(t)|}$$

rule: if a vector function has constant magnitude then it is perp. to the tangent vector.

why?



geometric proof

$B = T \times N$.

τ = torsion = how twisted the curve is = how fast B changes

$$\tau = -\frac{dB}{ds} \cdot N = \frac{(r' \times r'') \cdot r'''}{|r' \times r''|^2}$$

---

* The motion problem:

$r(t)$: position function

$$r'(t) = \lim_{h\to 0} \frac{r(t+h)-r(t)}{h} = \text{velocity (vector)} = v(t)$$

$|r'(t)| =$ speed (scalar)

$a(t) = r''(t) =$ acceleration

Ex

$$\begin{cases} x = \cos t \\ y = \sin t \\ z = \cos 2t \end{cases} \rightsquigarrow r(t) = \langle \cos t, \sin t, \cos 2t \rangle$$

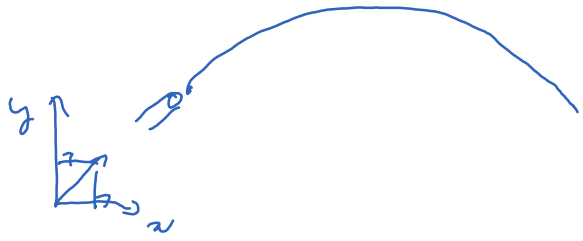
$r'(t) = \langle -\sin t, \cos t, -2\sin 2t \rangle$

$|r'(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (-2\sin 2t)^2} = \sqrt{1 + 4\sin^2 2t}$.

$\rightsquigarrow$ min when $\sin 2t = 0 \rightsquigarrow 2t = 0, \pi, 2\pi \rightsquigarrow t = 0, \frac{\pi}{2}, \pi$.

$(x, y, z) = (1, 0, 1), (0, 1, -1), (-1, 0, 1)$.

Ex: tennis ball launcher machine



$v_0 = 1$ m/s

angle $= 45°$

$m = 0.05$ kg

How far does the ball go?

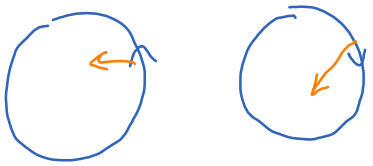
$F = ma$

$\langle 0, -mg \rangle = ma \rightsquigarrow a = \langle 0, -g \rangle$

$$v(t) = v(0) + \int_0^t a\,ds = \frac{v_0}{\sqrt{2}}\langle 1, 1 \rangle + \int_0^t \langle 0, -g \rangle\,ds$$

$$v(t) = \left\langle \frac{v_0}{\sqrt{2}}, \frac{v_0}{\sqrt{2}} \right\rangle + \langle t, -gt \rangle = \left\langle \frac{v_0}{\sqrt{2}} + t, \frac{v_0}{\sqrt{2}} - gt \right\rangle$$

$$r(t) = r(0) + \int_0^t v(s)\,ds = \langle 0, 0 \rangle + \int_0^t \left\langle \frac{v_0}{\sqrt{2}} + s, \frac{v_0}{\sqrt{2}} - gs \right\rangle ds = \dots$$



Acceleration, in general, isn't tangent to the trajectory. It points toward the "inside" of the trajectory.

$$a = a_T T + a_N N$$

$$a_T = v'(t)$$

$$a_N = k v(t)^2$$