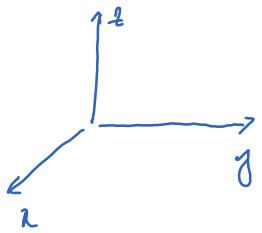
Lecture 2

Tuesday, January 4, 2022 8:20 PM

* Prayer
* Spiritual thought

3D space: distance between $P(a,b,c)$ and $Q(a',b',c')$ is

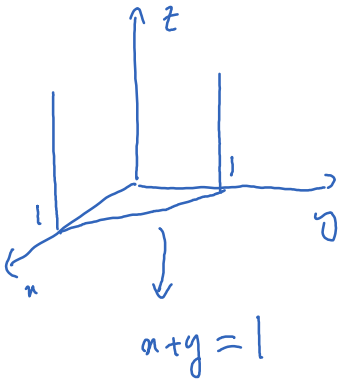
$$d = \sqrt{(a-a')^2+(b-b')^2+(c-c')^2}$$



sphere centered at $P(a,b,c)$ with radius r has equation

$$(x-a)^2+(y-b)^2+(z-c)^2 = r^2$$

plane: what is the equation of a plane?



Plane Oxy: $z=0$

Plane Oxz: $y=0$

Plane Oyz: $x=0$

In general: $ax+by+cz=d$

Ex Intersection of 3 objects:

plane 1: $x+y+z=0$ (1)

plane 2: $x-y+z=2$ (2)

sphere: $(x-2)^2+(y-1)^2+z^2=9$ (3)

Find the intersection:

$$\left.\begin{array}{l}(1)\rightsquigarrow \quad y+z=-x \\ (2)\rightsquigarrow \quad -y+z=2-x\end{array}\right\} \quad \begin{array}{l} z=1-x \\ y=-x-z=-x-1+x=-1\end{array}$$

$$(3)\rightsquigarrow \quad (x-2)^2+(-2)^2+(1-x)^2=9$$

$$\rightsquigarrow \quad (x-2)^2+(x-1)^2=5 \longrightarrow 2x^2-6x+5=5$$

$$\rightsquigarrow \quad x^2-3x=0$$

$$\rightsquigarrow \quad x=0 \text{ or } x=3$$

Plot on Mathematica:

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ContourPlot3D[{x + y + z == 0, x - y + z == 2, (x - 2)^2 + (y - 1)^2 + z^2
== 9}, {x, -5,5}, {y, -5,5}, {z, -5,5}, AxesLabel → Automatic]
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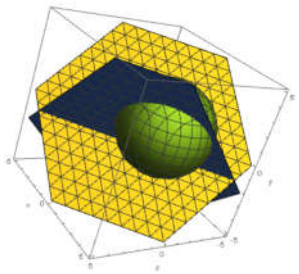
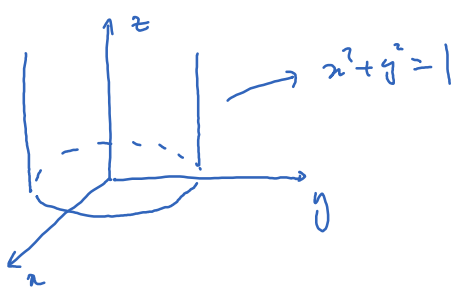
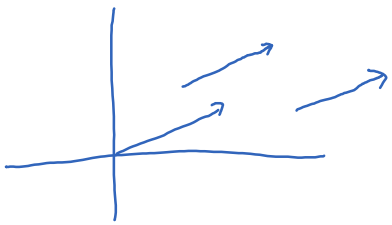


Cylinder:



Vectors



A vector has a dual nature:

Geometrically, vector is represented by an arrow.

Algebraically, vector (in 2D) is a pair of numbers.

Addition, scaling of vectors ....

length: $|v| = \sqrt{a^2+b^2}$

$v = \langle a, b \rangle$

Ex: $v = \langle 1, 2, 3 \rangle = i + 2j + 3k$

$w = \langle -2, 1, 1 \rangle = -2i + j + k$

What is $v + 2w$?

How about the angle between two vectors?

$$\cos\theta = \frac{v \cdot w}{|v||w|} \qquad 0 \le \theta \le \pi$$

This formula provides a simple way to check if two vectors are perpendicular to each other.