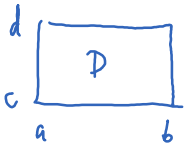
Lecture 23

Monday, February 28, 2022 11:00 AM

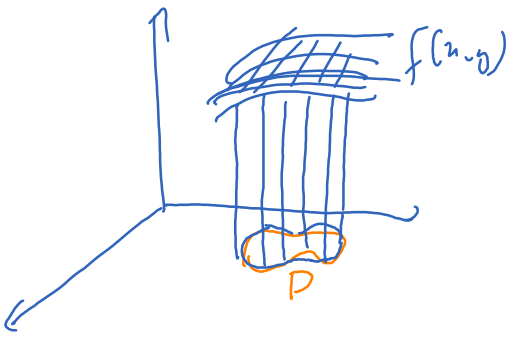
$$\iint_D f(x,y)\,dA$$

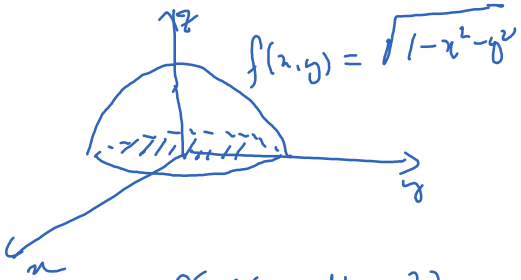


$$= \int_c^d \int_a^b f(x,y)\,dx\,dy$$

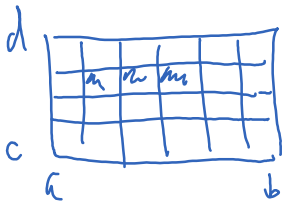
$$= \int_a^b \int_c^d f(x,y)\,dy\,dx$$

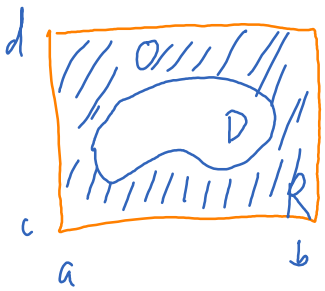
What if D is <u>not</u> a rectangle?





$\iint_{disk} f(x,y)\,dA$??

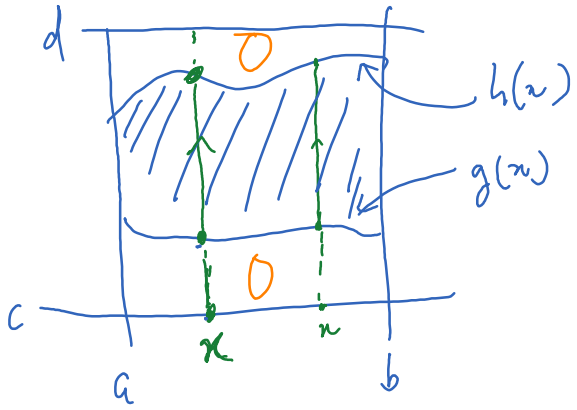




$$\iint_D f(x,y)\,dA \overset{\text{def}}{=} \iint_R f(x,y)\,dA$$

$$= \int_a^b \int_c^d f(x,y)\,dy\,dx$$

* Special cases of D :



$$D=\{(x,y)\mid a\le x\le b,\ g(x)\le y\le h(x)\}$$

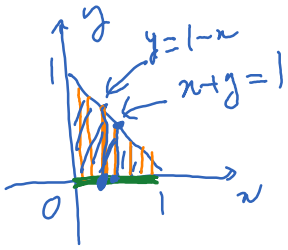
$$\iint_D f(x,y)\,dA$$

$$=\iint_R f(x,y)\,dA=\int_a^b\int_c^d f(x,y)\,dy\,dx$$

$$\iint_D f(x,y)\,dA=\int_a^b\int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$$

$$\int_{g(y)}^{h(y)}\int_a^b f(x,y)\,dx\,dy$$

Ex

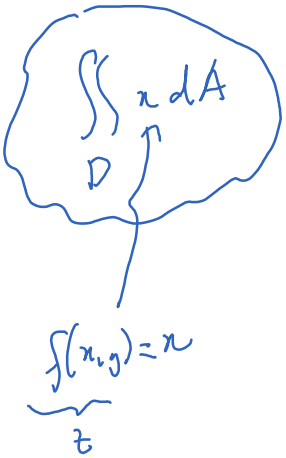


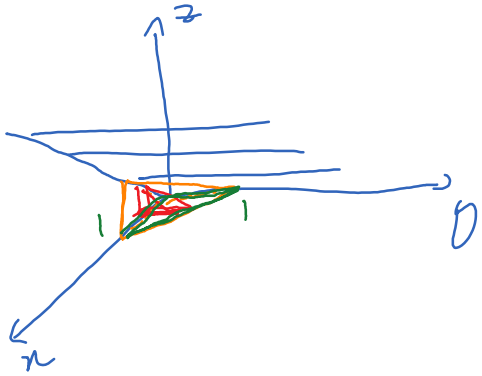
$$\iint_D x\,dA \qquad [f(x,y)=x]$$

$$D=\{(x,y)\mid 0\le x\le 1,\ 0\le y\le 1-x\}$$

$$\iint_D x\,dA=\int_0^1\int_0^{1-x} x\,dy\,dx=\int_0^1 xy\Big|_{y=0}^{y=1-x}dx$$

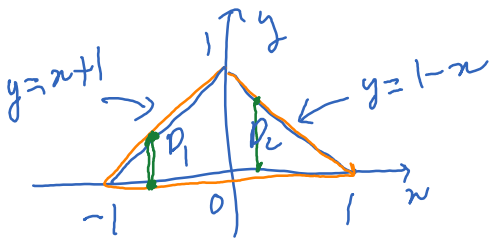
$$=\int_0^1 x(1-x)\,dx=\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_0^1=\frac{1}{2}$$





$$\iint_D x\,dA = \int_0^1 \int_0^{1-x} x\,dy\,dx$$

Ex

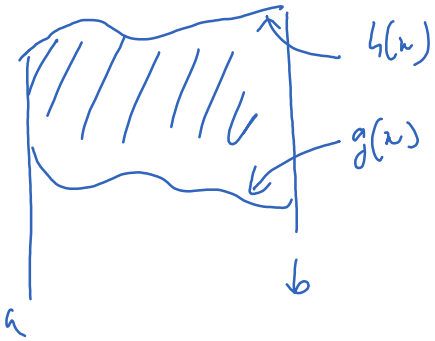


$$\iint_D x\,dA$$

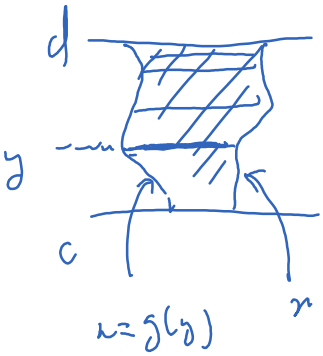
$$= \iint_{D_1} x\,dA + \iint_{D_2} x\,dA$$

$$= \int_{-1}^{0}\int_{0}^{x+1} x\,dy\,dx + \int_{0}^{1}\int_{0}^{1-x} x\,dy\,dx \Rightarrow$$

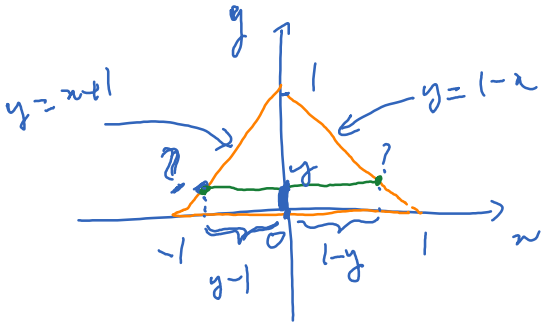
$$= \cdots$$



$$\iint_D = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$$



$$\iint_D f(x,y)\,dA = \int_c^d \int_{g(y)}^{h(y)} f(x,y)\,dx\,dy$$



$$D=\{(x,y)\mid 0\le y\le 1,\ y-1\le x\le 1-y\}$$

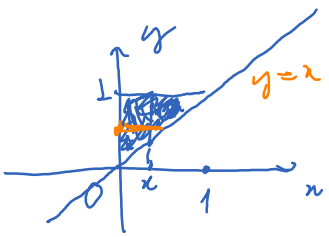
$$\iint_D x\,dA=\int_0^1\int_{y-1}^{1-y} x\,dx\,dy=\cdots$$

Ex

$$\int_0^1\int_x^1 \sin(y^2)\,dy\,dx=\iint_D \sin(y^2)\,dA=\int_0^1\int_0^y \sin(y^2)\,dx\,dy$$

$x\le y\le 1$

$0\le x\le 1$



$0\le y\le 1$

$0\le x\le y$

$$\int_0^1 y\sin(y^2)\,dy$$

u-substitution $u=y^2$ ....