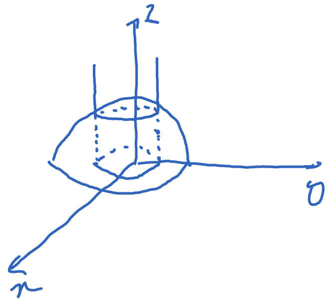
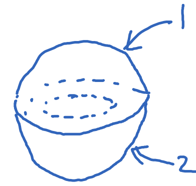
Lecture 25

Thursday, March 3, 2022 11:57 PM

* Prayer
* Spiritual thought

Problem 32 of 15-3

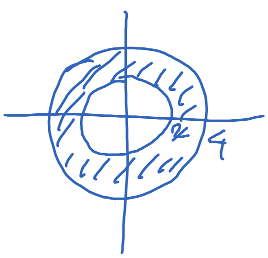




volume of solid under surface 1, above surface 2, on the annulus.

Surface 1: $f(x,y) = \sqrt{16-x^2-y^2}$

Surface 2: $-f(x,y) = -\sqrt{16-x^2-y^2}$



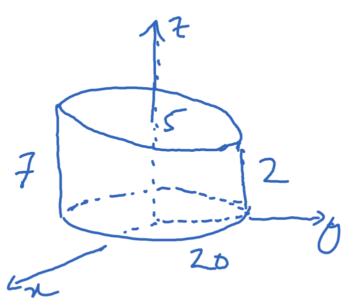
Annulus: $R = \{(r,\theta) \mid 2 \le r \le 4,\ 0 \le \theta \le 2\pi\}$

$$\text{volume} = \iint_R \left(f(x,y) - (-f(x,y))\right) dA$$

$$= \iint_R 2f(x,y)\, dA = \int_2^4 \int_0^{2\pi} 2\sqrt{16-r^2}\; r\, d\theta\, dr$$

$$= 2\pi \int_2^4 \sqrt{16-r^2}\; r\, dr = \dots$$

Problem 45 of 15-3:



Equation of surface:

$$\frac{z}{5} + \frac{y}{20} = 1$$

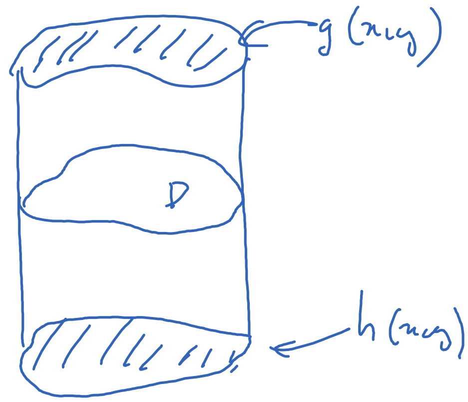
$$\longrightarrow z = 5 - \frac{y}{4}$$

$$\text{volume} = \iint_D \left(5 - \frac{y}{4}\right) dA$$

$$D = \{(r,\theta) \mid 0 \le r \le 20,\ 0 \le \theta \le 2\pi\}$$

$$\text{volume} = \int_0^{2\pi}\int_0^{20} \left(5 - \frac{r\sin\theta}{4}\right) r\,dr\,d\theta = \cdots$$

A step back: double integral can be used to find volume trapped between two surfaces restricted on a 2D region.



$$\text{vol} = \iint_D \left(g(x,y) - h(x,y)\right) dA$$

Imagine: finding the mass of a solid instead of the volume

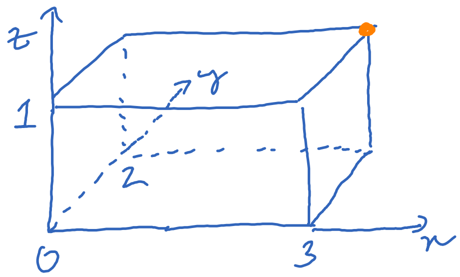
$f(x,y,z)$ = mass density (mass per unit volume)

$f(x,y,z)\,\underbrace{\Delta V}_{\text{small volume}} \approx$ mass of the material occupying ΔV

$\sum f(x,y,z)\,\Delta V \approx$ total mass

$\iiint_E f(x,y,z)\,dA$ = total mass $\rightsquigarrow$ triple integral

Ex



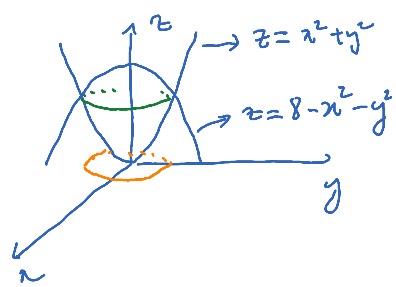
$E = [0,3] \times [0,2] \times [0,1]$

$f(x,y,z) = x+y+z$

The origin is where mass accumulates the least. The orange corner is where mass accumulates the most.

$$\text{mass} = \iiint_E (x+y+z)\,dV = \int_0^1\int_0^2\int_0^3 (x+y+z)\,dx\,dy\,dz$$

Ex

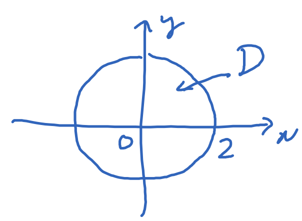


$$\iiint_E xz\,dV = ?$$

Domain of the form

$$E = \{(x,y,z) \mid (x,y) \in D,\ g(x,y) \le z \le h(x,y)\}$$

$$\iiint_E f(x,y,z)\,dV = \iint_D \int_{g(x,y)}^{h(x,y)} f(x,y,z)\,dz\,dA$$



In this case,

$$\iiint_E xz\,dV = \iint_D \int_{x^2+y^2}^{8-x^2-y^2} xz\,dz\,dA$$