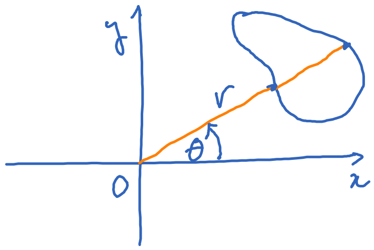
Lecture 26

Tuesday, March 8, 2022 10:52 PM

* Prayer

* Spiritual thought

Polar coordinates:



$$\iint_D f(x,y)\,dA = \int_{\theta_1}^{\theta_2}\int_{\phi_1(\theta)}^{\phi_2(\theta)} g(r,\theta)\,r\,dr\,d\theta$$

$$D = \{(r,\theta) \mid \phi_1(\theta) \le r \le \phi_2(\theta),\ \theta_1 \le \theta \le \theta_2\}$$

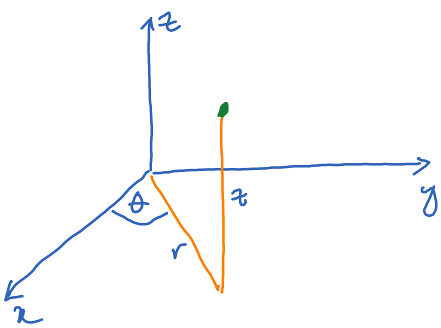
For certain regions, it is nicer to describe using polar coordinates than Cartesian coordinates.

Difficulty with double integral: region + integrand

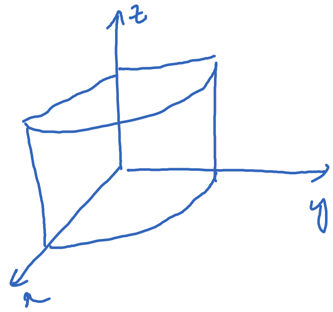
Using polar coordinates is equivalent to changing variables of the integral.

Each coordinate system corresponds to a way of scanning the region.

* Cylindrical coordinate system:



$$\underbrace{(x,y}_{\text{polar coord}},z) \xrightarrow{\text{cylindrical}} (\underbrace{r,\theta}, z)$$

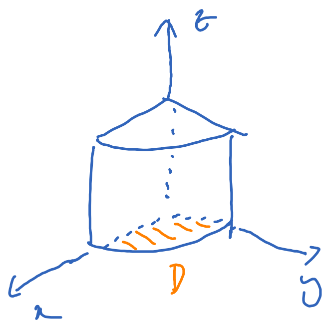


If solid E is described in terms of cylindrical coordinates as $E = \{(r,\theta,z) \mid \theta_1 \le \theta \le \theta_2,\ \phi_1(\theta) \le r \le \phi_2(\theta),\ g(r,\theta) \le z \le h(r,\theta)\}$

Then

$$\iiint_E f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2}\int_{\phi_1(\theta)}^{\phi_2(\theta)}\int_{g(r,\theta)}^{h(r,\theta)} \ldots\ldots\ r\,dz\,dr\,d\theta$$

Ex $\iiint_E z\,dV$ where E is the part of the circular cylinder $x^2+y^2=1$, bounded by $x=0$, $y=0$, $z=1$ and in the region where $x,y>0$.



$$\iiint_E z\,dV = \iint_D \int_0^1 z\,dz\,dA = \frac{1}{2}\iint_D dA$$

$$= \frac{1}{2}\,\text{area}(D) = \frac{1}{2}\,\frac{\pi}{2} = \frac{\pi}{4}.$$

Other regions that suite with cylindrical description: cone, circular cylinder