Lecture 28

Saturday, March 12, 2022 10:05 PM

* Prayer

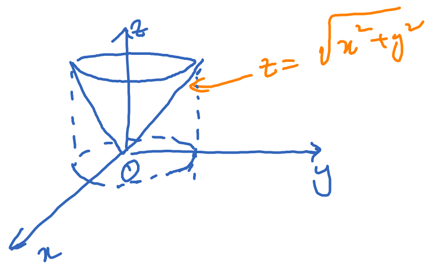
* Spiritual thought

* Cylindrical coordinates:

$$(x, y, z) \longrightarrow (r, \theta, z)$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad \frac{\partial(x,y,z)}{\partial(r,\theta,z)} = r$$

Ex

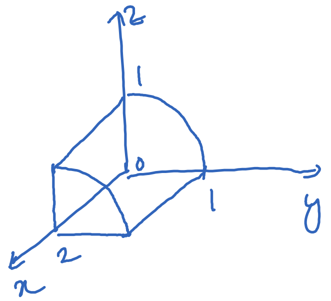


$$E: (r, \theta, z): \quad \underbrace{\begin{matrix} 0 \le r \le 1 \\ -\pi/2 \le \theta \le \pi/2 \\ 0 \le z \le r \end{matrix}}_{E'}$$

$$\iiint_E x\,dV = \iiint_{E'} r\cos\theta\, r\,dV'$$

$$= \underbrace{\int_0^1 \underbrace{\int_{-\pi/2}^{\pi/2} \underbrace{\int_0^r r\cos\theta\,dz}_{r^2\sin\theta}\,d\theta}_{2r^2}\,dr} = \int_0^1 2r^2\,dr = \frac{2}{3}$$

Ex



E: solid bounded by $x=0$, $x=2$, $y=0$, $z=0$, $y^2+z^2=1$

$$\iiint_E z\,dV = ?$$

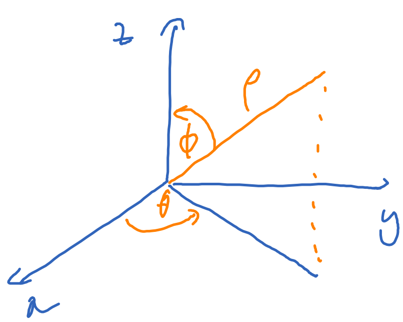
In cylindrical coords:

$$\begin{cases} x = x \\ y = r\sin\theta \\ z = r\cos\theta \end{cases} \qquad \left|\frac{\partial(x,y,z)}{\partial(x,r,\theta)}\right| = r$$

E: $0 \le x \le 2$, $0 \le r \le 1$, $0 \le \theta \le \frac{\pi}{2}$.

$$\iiint_E z\,dV = \iiint_{E'} r\cos\theta\, r\, dV' = \int_0^{\frac{\pi}{2}}\int_0^1\int_0^2 r^2\cos\theta\, dx\, dr\, d\theta = \cdots\cdots$$

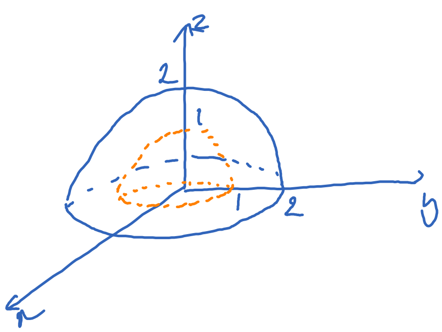
* Spherical coords



$$\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases} \qquad \rho > 0,\ 0 \le \phi \le \pi,\ 0 \le \theta \le 2\pi$$

$$\left|\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)}\right| = \rho^2\sin\phi$$

Ex



$$\text{vol} = \iiint_E 1\,dV$$

E: $1 \le \rho \le 2$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \frac{\pi}{2}$

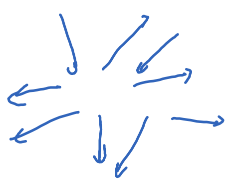
$$\text{vol} = \iiint_{E'} \rho^2 \sin\phi \, dV' = \int_1^2 \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho$$

$$= 2\pi \int_1^2 \int_0^{\frac{\pi}{2}} \rho^2 \sin\phi \, d\phi \, d\rho = 4\pi \int_1^2 \rho^2 \, d\rho = \frac{28\pi}{3}$$

Vector fields: draw vector field using Mathematica

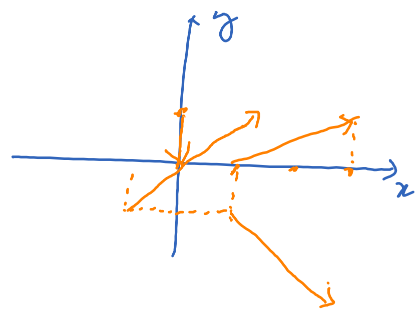
→ a map of arrows

An example of vector field is the gradient vector field.



Ex: $F(x,y) = \langle 2x, x-y \rangle$

x	y	$F(x,y)$
0	1	$\langle 0,-1 \rangle$
1	0	$\langle 2,1 \rangle$
1	-1	$\langle 2,2 \rangle$
-1	1	$\langle -2,-2 \rangle$
2	0	$\langle 4,2 \rangle$



On Mathematica:

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VectorPlot[{2x, x-y}, {x,-1,1}, {y,-1,1}]
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